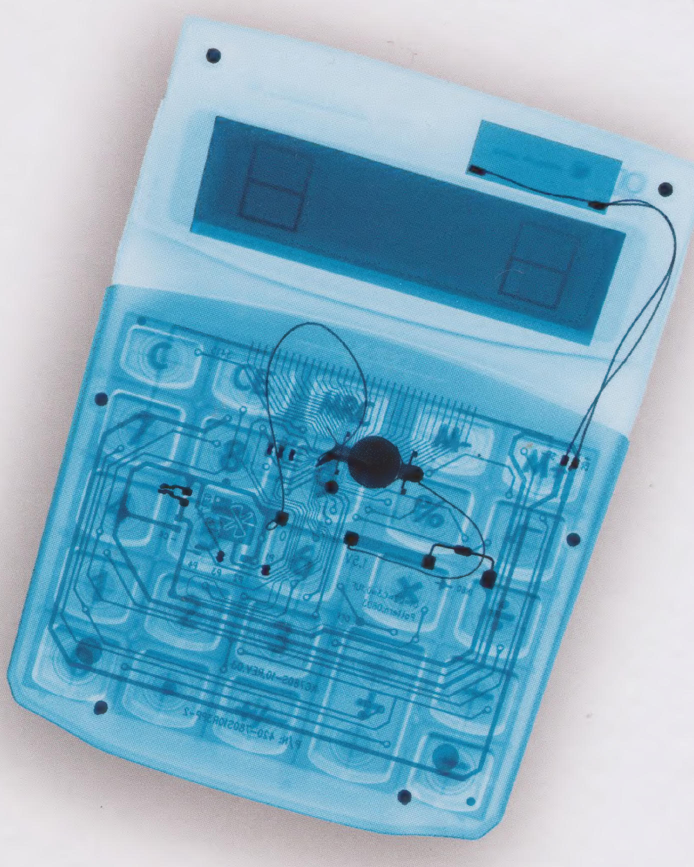
The Secret Life of Numbers

Mathematical curiosities



Everything is mathematical

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Joaquín Navarro

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Published by RBA Coleccionables, S.A.
c/o Hothouse Developments Ltd
91 Brick Lane, London, E1 6QL

Localisation: Windmill Books Ltd.
Photographs: Age Fotostock

ISSN: 2050-649X

Printed in Spain

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Preface

Mathematics is the music of reasoning.
James Joseph Sylvester

There are already collections of mathematical anecdotes published in various languages. The one which you have in your hand seeks to be something different, with lesser-told stories that are just as interesting in terms of the mathematics they contain. Many readers do not find mathematical anecdotes particularly amusing, and many mathematicians have said (and done) very serious things which are nevertheless worth retelling. It is true that some mathematical stories are not at all funny. In the last century the Nazi and communist regimes and others of totalitarian ideology forced mathematicians to live in situations that were about as much fun as, if you will excuse the directness, slitting your wrists, and many other events of that time are brimming with anecdotes that are either not very or not at all uplifting. There are other mathematicians, with lives full of curiosities, who have never knowingly told a joke in their lives. To tell the truth, few specialists in logic consider jokes to be simple accidents of language or compulsive objects of study.

There are immortal stories: the image of the computer genius Turing, for example, who may have committed suicide by biting into a poison apple in tragic imitation of Snow While; it is very moving and not in the slightest amusing. Nor is the tragic death from cancer of Ada Lovelace, whose condition was aggravated by her mother, who hid her morphine so that the pain would allow her to better atone her sins. Next to these two anecdotes, G.H. Hardy's amusing battle with God, the stormy sea and the Riemann hypothesis seems a trifle insignificant. Some mathematicians behave like serious people, while others do not.

In writing this book the hands of time have been selected as a marker, that is to say the stories are told according to the order in which they took place: first the oldest ones and then the more recent. And to make the reading more organised, each chapter is dedicated to a specific subject. The first, for example, is dedicated to stories related to the most basic element of mathematics, the numbers; the second to everything geometrical. The third covers stories about calculus. The fourth is a collection of curiosities related to all aspects of the mathematical disciplines and theories outside these fields up to the 20th century, which were well-known. Having covered all these specific subjects, the fifth and sixth chapters tell of mathematicians

who constitute, possibly to their detriment, a separate classification. The last chapter, as the grand finale, brings together unclassifiable things, such as horoscopes, in which many individuals, from many periods and origins, have been involved.

Mathematics is a science; perhaps, in the words of the historian E.T. Bell, the queen of the sciences. Its followers are somewhat unusual people, and its culture somewhat convoluted. It requires a certain clarity and depth of thought. After all, not everyone is a mathematician; if they were the world might be a very boring place. What this book modestly seeks is a slightly different approach to the usual history of mathematics, a less rigorous and scholarly approach; in short, a look at its hidden side.

Chapter 1

Numbers

Albert! Stop telling God what to do!
Niels Bohr to Albert Einstein

In the beginning there was number and form, and learning to master them is how science began, the comprehension of everything that surrounds us. Throughout this process, which will never end, amusing, intriguing and enlightening anecdotal events have occured. With no desire whatsoever to list all of them, or even the best-known ones, here are some of those which may be worth telling, for no reason other than to show the human side of a science – mathematics – that is all too often elevated to the category of divinity.

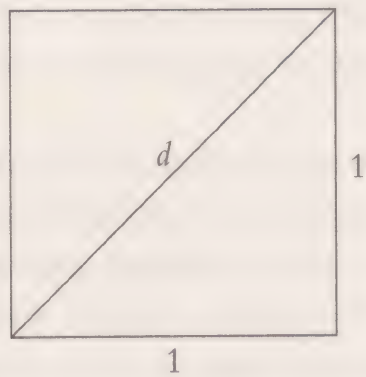
A great invention

Palamedes is a character from Greek mythology related to Agamemnon and Ulysses, heroes of the Trojan War. If there is one reason why he figures here it is because Plato mentions him, with a certain derision of mathematical origin. Palamedes is the supposed father, as legend has it, of weights and measurements and of their conceptual expression, the number. Palamedes invented the number – quite a contribution... Plato mocked the existence of Palamedes saying: "So Agamemnon, before speaking to Palamedes, didn't know how many feet he had?" The irreverent Plato, just as ungodly as his master Socrates, was ordered to commit suicide for his atheism.

The price of the truth

When a magnitude a was measured with a unit of measurement b, the ancient Greeks would say that the fraction a/b was the measurement of a. This is the same as saying that anything which is measurable has a fractional measurement or, in modern terms, all measurement is equivalent to a rational number and vice versa. For example, if a line was (in our eyes) 70 cm long and the ruler measured 20 cm,

to the Greeks, the fraction $70/20 = 7/2$ was the measurement of *a* carried out with *b*. That was the governing and unshakeable belief of the Pythagorean school and its members. Of all of them? Well, no, there was one black sheep: Hippasus of Metapontum (5th century BC) discovered, it seems, that if he tried to measure the diagonal of a square with unit sides, the measurement was impossible; not just difficult, but impossible.



Incommensurable diagonal of a square.

If $d = a/b$, it is evident that *a* and *b* can be coprime numbers. All we have to do is simplify the fraction a/b. Now we take, in order to reduce it to its simplest expression, the unit square. The theorem of Pythagoras himself tells us that $d^2 = 1^2 + 1^2 = 1 + 1 = 2$, that is $(a/b)^2 = 2$ or, if you prefer, $a^2 = 2b^2$.

Let's take a look at *a*: if it is even, *b* must be odd, as we have supposed that *a* and *b* are coprime. As $a = 2p$, the equality gives us $(2p)^2 = 4p^2 = 2b^2$ and, consequently, $2p^2 = b^2$, from which it can be deduced that b^2 and, therefore, *b*, must be even. Which is impossible, as we have already seen that *b* is odd.

Now, let's assume that *a* is odd. But then a^2 is also odd. But $a^2 = 2b^2$, which means that a^2 is even, which flagrantly contradicts what we have assumed. Something unthinkable had been demonstrated and Hippasus, a Pythagorean, a member of the sect, had proved it. And, of course, the best solution when receiving an unacceptable message is to kill the messenger.

Iamblichus of Chalcis stated, eight centuries later, that the Pythagoreans built a tomb for the prophet of the incommensurable; it must have been a bad omen. Actually, there is more than one version of Hippasus' fate; the least brutal version does not even mention Hippasus and only goes as far as explaining that the Pythagoreans sacrificed one hundred oxen in their astonishment at such

incommensurability. Given that the Pythagoreans were vegetarians, this bovine hecatomb seems possible, but not particularly credible. A variant of this version states that Hippasus was expelled from the Pythagorean sect. The gory version describes how he was executed by throwing him over the side of a boat. Whatever actually happened, the Pythagoreans stubbornly stuck to their beliefs. It was not until Eudoxus of Cnidus and his *de facto* introduction of real numbers that the enigma of incommensurability was solved.

The Holy Gospels, the fish and 153

One of the most ancient mentions of numbers in Western history can be found in the Gospel of St. John, 21, where he tells of Simon Peter's extraordinary fishing achievement, in which his net traps 153 fish in one go on the Sea of Galilee – of course the church accepts that it was a miracle of Jesus Christ.

One hundred and fifty three is a number which is bound to have repercussions, and the first thing we discover is that it is a triangular number. Count the asterisks if you like, there are, indeed, 153:

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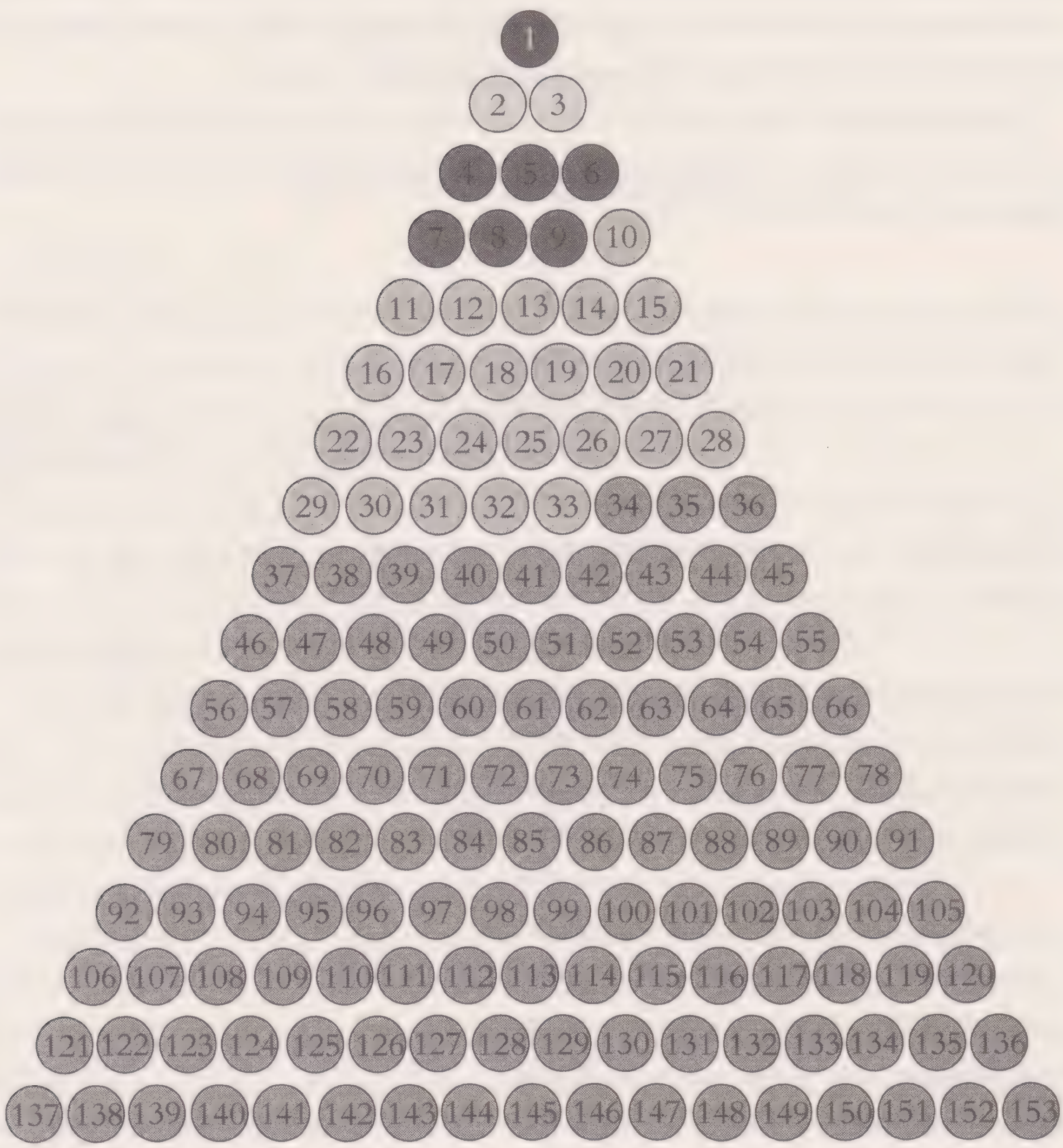
However, although 153 is a triangular number, that is not reason enough for it to appear in the Gospels; something more is required. If we take:

$$1! = 1$$
$$2! = 2 \cdot 1 = 2$$
$$3! = 3 \cdot 2 \cdot 1 = 6$$
$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$
$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120,$$

we can see that $1! + 2! + 3! + 4! + 5! = 1 + 2 + 6 + 24 + 120 = 153$, as shown in this diagram:



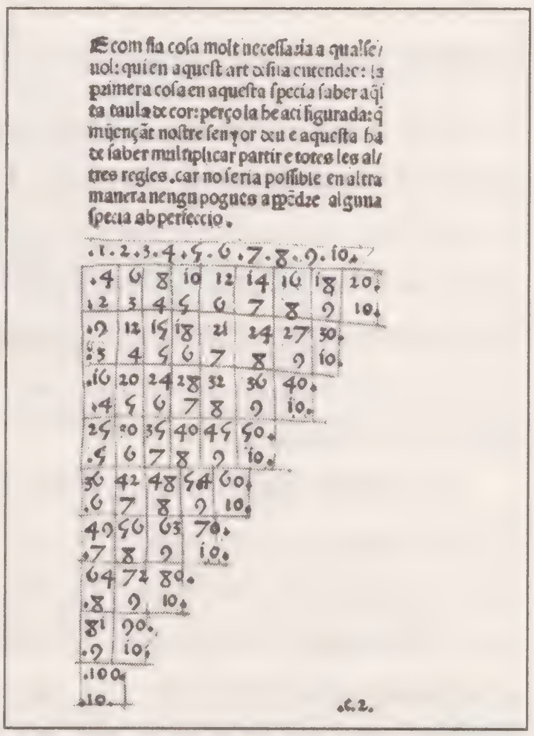
This is a step forward, but there will still be unbelievers who consider all this unworthy of building a divine mathematical anecdote around. If we search even deeper, we find the following. Given that the Lord is triune, let's take any multiple of three, for example 1,728 (any multiple of three would do), raise its figures to three and add them, repeatedly:

$$1^3+7^3+2^3+8^3=864$$
$$8^3+6^3+4^3=792$$
$$7^3+9^3+2^3=1{,}080$$
$$1^3+0^3+8^3+0^3=513$$
$$5^3+1^3+3^3=153.$$

And there we have it, the series always finishes on 153. Try it yourself with any multiple of three other than 1,728. Miracle or mathematics?

Tradesmen before mathematicians

This was certainly the case during the Renaissance. In 1456 printing was invented and books began to travel the world, opening the minds of many to the winds of progress. Many minds, but not all of them, or at least not ours, in the comfortable watchtower of the 21st century, would have liked this. For example, the first printed mathematics text in history was not (even given the difficulty of translating it and its considerable length) Euclid's *Elements of Geometry* as you might expect since it is a genuine monument to the mathematical wisdom of the ancient world. Instead, the first printed mathematics book was *Treviso Arithmetic*, or *L'arte de l'abbacho* to give it its original title, an elementary academic treatise which only applied the four basic operations and proposed practical exercises on proportional distributions. It was published in 1478 and used Indo-Arabic numbers.



A page from Treviso Arithmetic.

The merchants who were the natural readers of such books prevailed over the scholars and thinkers. The future, however, would take its revenge. There was never even a second edition of *Arithmetic*. There were hundreds of Euclid's *Elements*.

The letters ran out

A story, which undoubtedly straddles the boundary between legend and reality, explains why the letter x dominates algebraic expressions in analytical geometry and, generally, in all mathematical texts. Normally we follow the route of René Descartes (1596-1650) in his book *The Geometry* (and that of his peers) and assign the first letters of the alphabet ($a, b, c, d, \ldots$) to known numerical entities. It is also customary to represent unknown magnitudes with the letters x, y, z, the last letters of the alphabet. Among them, x is the queen, the letter which represents the unknowns par excellence.

According to some, the reason for this is very prosaic. The book's typographer realised that he would be missing type for other letters, while he had a good stock of little-used x letters; so when he had to assign a symbol to the unknown, he used x. What we do know is that Descartes' notation set a trend, and since then the world has followed.

Leibniz and the Emperor of China

To thinker and polymath Gottfried Wilhelm Leibniz (1646-1716) the binary system provided a sudden enlightenment. Discovering the kingdom of the 1s and 0s was to him like discovering the transmutation of lead into gold. All of a sudden new doors were opened, some of which were absurd. One (which was like God) and zero (the abyss), could explain the entire Universe, just as the simple 1 and the simple 0 could generate all of the numbers. This had to be explained, and such a wonder had to be exploited.

In 1689, Leibniz contacted his Jesuit friend Carlo-Filippo Grimaldi, president of the tribunal of mathematics in China (in subsequent years they would have interesting correspondence). He told him everything he thought and asked him to use his influence and powers of persuasion to convince emperor Cam–Hi that his ideas were good. After all, were the Chinese not the creators of ying and yang? The emperor of China should abandon Buddhism and embrace Christianity shortly after gaining a little knowledge of the 1 and the 0. None of this happened. The

emperor did not become a Christian, Leibniz regretted it and the binary numbering system remained relegated to the kingdom of arithmetic, which it should never have left.

Leibniz nevertheless fanatically attributed semi-divine characteristics to new mathematical entities with which he was presented. He considered imaginary numbers, those linked to the mysterious $\sqrt{-1}$, to be sublime and magnificent, "amphibians between being and non-being".

Damn kid

An anecdote from the childhood of Carl Friedrich Gauss (1777-1855) is often told, which portrays his precocious personality. He was 10 years old and his teacher, no doubt seeking a few minutes of peace and quiet in the classroom, set him and his classmates a task that would definitely take them a while: adding all the numbers from 1 to 100:

$$1+2+3+\ldots+98+99+100.$$

After a few minutes the young Gauss stood up and handed the teacher the result, written on his mini-blackboard: 5,050. How was this little 'monster' of a kid able to solve it? We now think that Gauss realised that by writing the succession forwards and backwards,

$$\begin{gathered}1+2+3+\ldots+98+99+100\\100+99+98+\ldots+3+2+1,\end{gathered}$$

he obtained partial sums for each pair, all of them of the same value:

$$1+100=2+99=3+98=\ldots=98+3=99+2=100+1=101.$$

How many sums were there? 100. And as that was double the total sum which the teacher wanted, the result must have been:

$$\frac{100\cdot 101}{2}=50\cdot 101=5{,}050.$$

So far, you have read of the legend of a child with a fabulous gift for reasoning. But the anecdote is normally told in simplified form, so that everyone can understand

it (not everybody is a clever as Gauss), and the original problem was even more overwhelming, as what the teacher actually proposed to the class was the sum of the first 100 terms in the series:

$$81{,}297 + 81{,}495 + 81{,}693 + \ldots,$$

where each term is 198 more than the previous one. The result is not as simple as before; Gauss was even more intelligent than legend has conveyed.

Fermat and Kummer

In 1847, the French mathematician Gabriel Lamé (1795-1870), in the presence of many of his colleagues, announced very excitedly that he had proved what we now know about the Fermat theorem. Lamé, who was a person of integrity, did not forget to add that he owed gratitude and inspiration to his colleague Joseph Liouville (1809-1882), who was also present, without whose help and inestimable collaboration he would not... etc. etc.

Attention then turned to Liouville, who thought all this was great, but he warned those present of a small detail: Lamé's demonstration was valid as long as unique factorisation was valid between a certain class of integers (which we will define later), such as the known integers. It should be said that few had their doubts. Lamé tried to demonstrate the small detail which was missing, but, to his desperation, there was no way to do so. As a music critic once said of a Debussy piece, the music was not very noisy, but it was a very unpleasant noise. Lamé was losing patience over a triviality.

Three years before, the German mathematician Ernst Kummer (1810-1893) had already published (in a cult German magazine) a counterexample of 'non-unique' factorisation between a certain class of numbers. When he heard about Lamé's efforts, he rushed to send him the counterexample. Lamé, disheartened, abandoned his attempted demonstration.

Lamé's famous integers are today called quadratic integers and they were objects which had been little studied at the time. Among the normal integers, such as those of $\mathbb{Z}$, there is unique factorisation (except for the presence of the units 1 or -1). For example,

$$6 = 2 \cdot 3 = 2 \cdot (-3) \cdot (-1) = (-2) \cdot 3 \cdot (-1) = (-2) \cdot (-3),$$

and, except for the presence of units, the factors are 2 and 3. On the other hand, in $\mathbb{Z}[\sqrt{-5}]$ (numbers of the type $a+ib\sqrt{5}$, where a and b are integers), we get, apart from 1,

$$6=2\cdot 3=(1+i\sqrt{5})\cdot(1-i\sqrt{5}),$$

and the factors are no longer unique. For example, the integer 6 factorises in two different ways (excluding 1).

Every cloud has a silver lining, as the saying goes. And, indeed, Kummer unleashed the hunt for Fermat's theorem, the famous and unproven conjecture

"There is no integer triple x, y, x which satisfies $x^n+y^n=z^n$, for $n > 2$"

was later proven for the first 100 exponents ($n \leq 100$). So, that only left an infinity more.

Ernst Kummer.

As well as a numerologist, Ernst Kummer was a fervent patriot, and was famous for his difficulty in remembering the basics of elemental arithmetic; the simple multiplication tables were a mystery to Kummer. When he needed them in class, he would turn to his students; for example: "Seven times nine, ummmm...", a perverse student suggests an incorrect answer, "seven times nine, sixty one". "No, no, sixty nine" suggests another, joining in the fun. And the poor Kummer says innocently

"Come on gentlemen, it has to be one or the other". And so Kummer set about some rational thinking. The intriguing thing is the solution to the enigma of $7 \cdot 9$, in pure Kummer style: 60, 62, 64, 66 and 68 are no good because they are even; 61 and 67 are discarded as they are prime numbers, and 65 because it ends in 5 and it is, therefore, a multiple of 5. It cannot be 69 as it is obviously too large. That leaves 63, and that must be the answer. Ergo $7 \cdot 9 = 63$. Pure Kummerian reasoning without knowing the seven times table.

1 + 1 = 2 and other basic equalities

German mathematician Peter Gustav Lejeune Dirichlet (1805-1859) had a special feeling for numbers; it is said that he would even be lulled to sleep by a volume of *Disquisitione arithmeticae* by Gauss, which he always placed under his pillow. Dirichlet made unusual use of numbers and technique. When his first child was born, the telegram received by his father-in-law said:

$$2 + 1 = 3.$$

It could not be clearer: before there were two and a third was coming. Moreover, at a time when telegrams were expensive, this idea of Dirilicht's had the virtue of brevity... and therefore it was cheap. He was not the first or the last person in history to utilize a formula as a statement: Socrates himself was pondering the equation '$1 + 1 = 2$' without being able to convince himself of its proof; but what should we expect from someone whose maxim is "I only know that I know nothing"?

Austrian physicist and mathematician Ludwig Boltzmann (1844-1906) played out an amusing scene in front of his students. As he was quick at mental arithmetic, his classes sometimes became torturous for attendees, because he skipped a lot of steps, assuming that all the reasoning and calculation going on in his head, which he did not even show on the board, were completely obvious to everyone who was watching. He was told in a friendly way that this was not the case, and Boltzmann, obediently promised to fix the problem. And he continued with the demonstration: "As I was saying, as $pv = p_0 v_0 (1 + \alpha t)$, etc., etc.", but, as usual, without making any notes on the board. Even worse than before. The unintelligible class ended with the immortal phrase: "I trust that everything I have said is as evident to you as one plus one is two." And, to round things off, to back up his incomprehensible lecture and suddenly remembering his promise to write down all his calculations, he walked over to the blackboard and conscientiously wrote: "$1 + 1 = 2$"!

A little later, Bertrand Russell (1872-1970) and Alfred North Whitehead (1861-1947) astonished the scientific world when, at the dawn of the 20th century (1910-1913), they unleashed a treatise of three volumes of abstruse, almost impossible, reading which they called – emulating Newton – *Principia Mathematica*. The equality in the title of this section, "1 + 1 = 2", which will seem evident to the uninitiated, is theorem number 54.43. It is in the second volume; the first sets the scene, so to speak. We can get an idea of how fun and easy it is to get into *Principia Mathematica's* intricate symbolic journeys from the fact that a respectable journalist of the time instituted a prize for anyone who could prove they had read the whole book. The competition was declared void. For a while there was hope that at least one of the co-authors, Whitehead, would be able to win the prize – but in vain. Each author had read only their part.

***54·43.** $\vdash :. \alpha, \beta \in 1 \,.\, \supset : \alpha \cap \beta = \Lambda \,.\, \equiv .\, \alpha \cup \beta \in 2$

Dem.

$$\vdash .\, {*}54{\cdot}26 \,.\, \supset \vdash :. \alpha = \iota`x \,.\, \beta = \iota`y \,.\, \supset : \alpha \cup \beta \in 2 \,.\, \equiv .\, x \neq y \,.$$

$$[{*}51{\cdot}231] \qquad \equiv .\, \iota`x \cap \iota`y = \Lambda \,.$$

$$[{*}13{\cdot}12] \qquad \equiv .\, \alpha \cap \beta = \Lambda \qquad (1)$$

$$\vdash .\, (1) \,.\, {*}11{\cdot}11{\cdot}35 \,.\, \supset$$

$$\vdash :. (\exists x, y) \,.\, \alpha = \iota`x \,.\, \beta = \iota`y \,.\, \supset : \alpha \cup \beta \in 2 \,.\, \equiv .\, \alpha \cap \beta = \Lambda \qquad (2)$$

$$\vdash .\, (2) \,.\, {*}11{\cdot}54 \,.\, {*}52{\cdot}1 \,.\, \supset \vdash .\, \text{Prop}$$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Part of the Principia Mathematica *which contains the demonstration that $1 + 1 = 2$. But first, as we are charmingly and ironically reminded by the text (the mischievous hand of Russell is evident), arithmetic addition must be defined.*

Small errors

Augustin Louis Cauchy (1789-1857) once received an intelligently written document on the theory of numbers which demonstrated that the impressive Diophantine equation

$$x^3 + y^3 + z^3 = t^3$$

had no integer solutions. Cauchy, who seems to have hidden a somewhat sarcastic and even cheerful character behind his ultramontane beliefs, answered by returning the original with a simple one-line note:

$$3^3 + 4^3 + 5^3 = 6^3.$$

Something similar happened to Alphonse de Polignac (1817-1890), an excellent French mathematician, quite well known today for a conjecture on the prime numbers which generalises the popular Goldbach conjecture. Polignac stated that:

"Every odd number is the sum of a prime and a power of 2."

It sounds impressive, and at a glance, credible; if you take any odd number, 63 for example

$$63 = 2^5 + 31,$$

and as 31 is a prime number, it seems to work. If we add to this evidence the fact that Polignac led people to believe that he had tested his conjecture up to the number 3,000,000, it is understandable that many people took an interest in the matter. However, if you take a number as small as 127, the conjecture does not work. Indeed, if you make a list of the six possible powers of two:

$$\begin{gathered}127 = 2^1 + 125 = 2^1 + 5 \cdot 25;\\127 = 2^2 + 123 = 2^2 + 3 \cdot 41;\\127 = 2^3 + 119 = 2^3 + 7 \cdot 17;\\127 = 2^4 + 111 = 2^4 + 3 \cdot 37;\\127 = 2^5 + 95 = 2^5 + 5 \cdot 19;\\127 = 2^6 + 63 = 2^6 + 3 \cdot 21,\end{gathered}$$

the following power of 2 is $2^8 = 128$, which is greater than 127. So, the fact is that the conjecture does not work for 127, even if Polignac did conjecture it. Some errors are fatal.

An astonishing calculation

This story is completely true and it is well documented. It takes place during an American Mathematical Society meeting in 1903. It was announced that the numerologist Frank Nelson Cole (1861-1926) would give a talk titled *On the Factorisation of Large Numbers*. The way in which the meeting unfolded was a little unusual: Cole got out of his seat, walked over to the blackboard and wrote: $2^{67}-1$, the Mersenne number M_{67}, supposedly (until then) a prime number. Cole dramatically went on to calculate 2^{67} and subtracted 1 from it. Then, while his public held their breath,

Cole wrote two numbers and he multiplied them by hand, on the blackboard: $193707721 \cdot 761838257287$. The result, 147573952589676412927 was, as everyone was expecting, the desired M_{67}. Cole then turned on his heel and went back to his seat. During the long hour which all this lasted, he did not mutter a word. The distinguished and wise audience broke into applause at the end, as if it were an opera.

It should be noted that in 1903 there were no calculators or algorithms capable of taking on Mersenne numbers like there are today. According to Cole, his incredible calculation had taken him "three years of Sundays". As a result of such an achievement, the American Mathematical Society instituted the now highly valued Cole prize. The hunt for Mersenne prime numbers can be followed live on the Internet, above all on the Great Internet Mersenne Prime Search website (http://www.mersenne.org/default.php). The largest known prime number in 2013 was $M_{43,112,609}$, a large number (truly large) with 12,978,189 digits. Ah, and $M_{43,112,609}$ starts with 3 and ends in 1. Please, don't ask for more.

A very large number

In mathematics we can think of arbitrarily large numbers that are finite, but very large, enormous, even colossal. In 1938, the nine-year old nephew of the well-known mathematician Edward Kasner (1878-1955) invented the 'googol' for his uncle's use, which was to his young mind an almost inconceivably large number, almost infinity. For Milton Sirotta, which is the name of the nephew, a googol was 1 followed by 100 zeroes. Written in 'adult' arithmetic

$$1 \text{ googol} = 10^{100}.$$

Considering its size, the googol does little to impress; the so-called 'googolplex', which is a 1 followed by a googol of zeroes is much more effective. We can also write this number now in 'adult' notation:

$$1 \text{ googolplex} = 10^{1 \text{ googol}} = 10^{10^{100}}.$$

For many years Sirotta's ingenius invention was included in all the mathematical textbooks as a clever idea until the arrival of Google. The now giant company was formed in 1998 by two young US mathematicians, Larry Page (b. 1973) and Sergey Brin (b. 1973). It began by offering a simple 'search engine', a great contribution to the Internet world which has been followed by many other tools. The name of the company is nothing more than a variant of the word 'googol', a term which, as we

have seen, had already been used copiously. When Google was created there were no more than 24 million indexed web sites, a number which is a long way from a googol of pages, but it is a well known fact that mathematicians are optimists.

The saga of 1,729

The reason for the mysticism surrounding the figure 1,729 lies, above all, in a well-known and repeated anecdote associated with two mathematicians, Englishman Godfrey Harold Hardy (1877-1947) and Indian Srinivasa Ramanujan (1887-1920). Hardy tells us that, in a visit to the then hospitalised Ramanujan, he took taxi number 1729, and he told Ramanujan frivolously in order to raise his spirits. According to Hardy, it was a dull number. "No", responded the Indian, "it is the smallest number that can be expressed as the sum of two cubes in two different ways". And, indeed,

$$1{,}729 = 12^3 + 1^3 = 9^3 + 10^3.$$

To demonstrate what Ramanujan had said off-hand and with an air of disinterest took Hardy two weeks. Subsequently, 1,729 has given rise to a sub-theory of the theory of numbers – the 'taxicab numbers' – but that is another story.

The anecdote, although very well known, is magnificent and true and gives an idea of how the mind of someone like Ramanujan must work. However, the continuation of the anecdote is equally intriguing and affects another physics and mathematics genius, Nobel prize for Physics winner Richard Feynman (1918-1988).

According to Feynman himself in *Surely You're Joking, Mr. Feynman!* 1,729 allowed him to beat a Japanese abacus operator, which is a remarkable performance because oriental abacus operators can be very good calculators. Seeing that the results with pencil and paper were improving as the calculations became more complex, the abacus operator proposed moving on to the difficult field of cubic roots, and made the error of asking Feynman to choose a number to find the cubic root of. Feynman immediately chose 1,729, a number which did not arouse any suspicion, but

$$\sqrt[3]{1{,}729} = \sqrt[3]{12^3 + 1^3} = \sqrt[3]{12^3\left(1 + \frac{1}{12^3}\right)} = 12\sqrt[3]{1 + \frac{1}{1{,}728}},$$

which can be easily written on paper, and applying a development of the Taylor series:

$$\sqrt[3]{1+\frac{1}{1,728}}=1+\frac{1}{3}\frac{1}{1,728}+\dots$$

These terms are enough to ensure (and Feynman was excellent at mental arithmetic) that

$$\sqrt[3]{1,729}\approx 12.0023,$$

a result which beat the abacus. Such that Feynman won his competition and Ramanujan symbolically blessed him from heaven, nirvana or whichever sacred place he may rest in.

An Indian stamp dedicated to Srinivasa Ramanujan, the greatest Indian mathematician in history.

Hardy, God and the Riemann hypothesis

Many anecdotes have been told about the quintessentially English mathematician and writer G.H. Hardy, of which one of the best-known is the following. He was an attractive personality, and a list of his fervent ambitions (confessed by Hardy himself) clearly explains why. Hardy's ambitions, over and above other mundane vanities, were:

1. To prove the Riemann hypothesis.
2. To play a brilliant innings in a crucial cricket match.
3. To murder Mussolini.
4. To demonstrate the non-existence of God.

The story that follows is related to the first of these wishes. The premises that need to be understood in order to understand why such a well-known anecdote appears in this book are:

— G.H. Hardy, a magnificent numerologist, known above all, as we have seen, for being ultimately responsible for bringing a character as extraordinary as Srinivasa Ramanujan to the West.
— God, who needs no introduction and whom Hardy considered his personal enemy.
— The Riemann hypothesis, without doubt the most important current and unproven conjecture in mathematics.

We are going to limit ourselves to the version of events told by George Pólya (1887–1985), as it reveals Hardy's ability and the mathematical reasoning which followed. Hardy was on his way back from Denmark, where he had visited Harald Bohr, mathematician and brother of the famous physicist Niels Bohr. Upon his return to England by boat the weather was bad, and the risk of the boat sinking was greater than normal. So Hardy sent Bohr a postcard containing the following message: "I have proved the Riemann hypothesis". Now let's take a look at the reasoning behind these events. If the boat had sunk, the whole world would think (through Bohr, a citizen entirely above suspicion) that Hardy had proved the Riemann hypothesis. But God could not allow a faithless enemy like Hardy the glory of such a discovery and, therefore, he would not let the boat sink. Ergo, the boat could not sink. *Quod erat demonstrandum.* Needless to say, Hardy, thank God, returned to England safely.

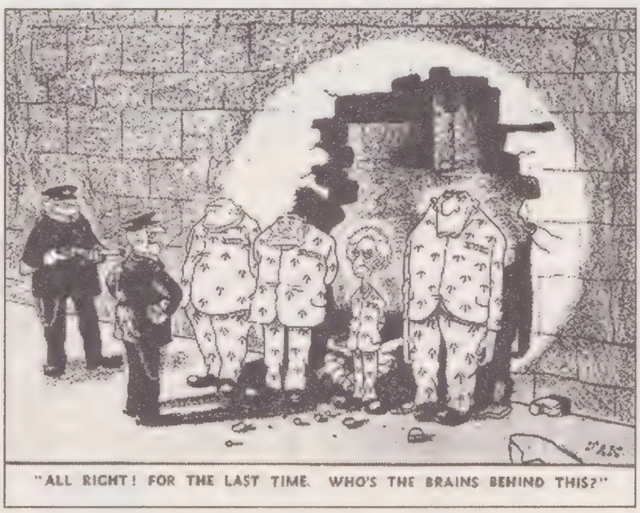
Such reasoning can become fallacious if not applied with care. We often hear the story of a statistician who calculated the probability of someone carrying an explosive device in their suitcase on a flight. When he realised that the probability of catastrophe on the flight was much reduced if there were two devices, he always travelled with a bomb in his suitcase. According to him, the probability of suffering an air accident was far smaller than if there was only one bomb on the plane. Of course, this is ridiculous and not a valid statistical deduction.

Another variation of the Hardy story is attributed to David Hilbert (1862-1943), although the protagonist is now Fermat's theorem (a mere conjecture at the time). This time, Hilbert had notified his colleagues from a far-off city where he was to announce in a conference that he would talk there about an eventual demonstration of the conjecture. When Hilbert, after an uneventful flight, gave his lecture, which was

brilliant as his always were, he did not actually mention the conjecture or Fermat at all. When they asked him about it he simply said that the Fermat thing "was just in case the plane blew up". God does not appear in this tale, but his role is evident.

Zero and nothing

During an interview between expert thinker Bertrand Russell and the Indian writer R.K. Laxman (b. 1924), who retold the story for posterity, the verbose Russell remarked in the middle of the conversation that India had not invented anything: *"You Indians, have invented absolutely nothing"*, Russell said. Laxman understood, with surprise, that the Indians could not claim to have contributed anything to the heritage of world inventions. We can assume that Laxman was somewhat astonished. Not only did it appear to be discourteous of Russell, which would be unimaginable from a gentleman such as he, but also a statement which did not reflect the truth. But he had not time to protest. The devious Russell clarified that the word *nothing* should be understood literally. *Nothing* meant 'zero', and that is exactly what Russell meant. The Indian culture had invented the zero.



Caricature from the Evening Standard *in 1961, one of the occasions on which Russell was imprisoned for defending his political principals, which were illegal at the time.*

Whether it was the Indians or someone else (it seems reasonable to attribute the origins of its use to 6th century Indians), whoever invented the zero not only invented a way of writing 'nothing', but also something very large. The concept of zero is at the very basis of positional numeration in arithmetic. Bertrand Russell was an earl, a Nobel prize winner and one of the most famous and intelligent

mathematicians in history. Even so he did not invent anything like the zero, an idea which is as ingenious as the wheel, silicon transistor or agriculture.

An unusual genius

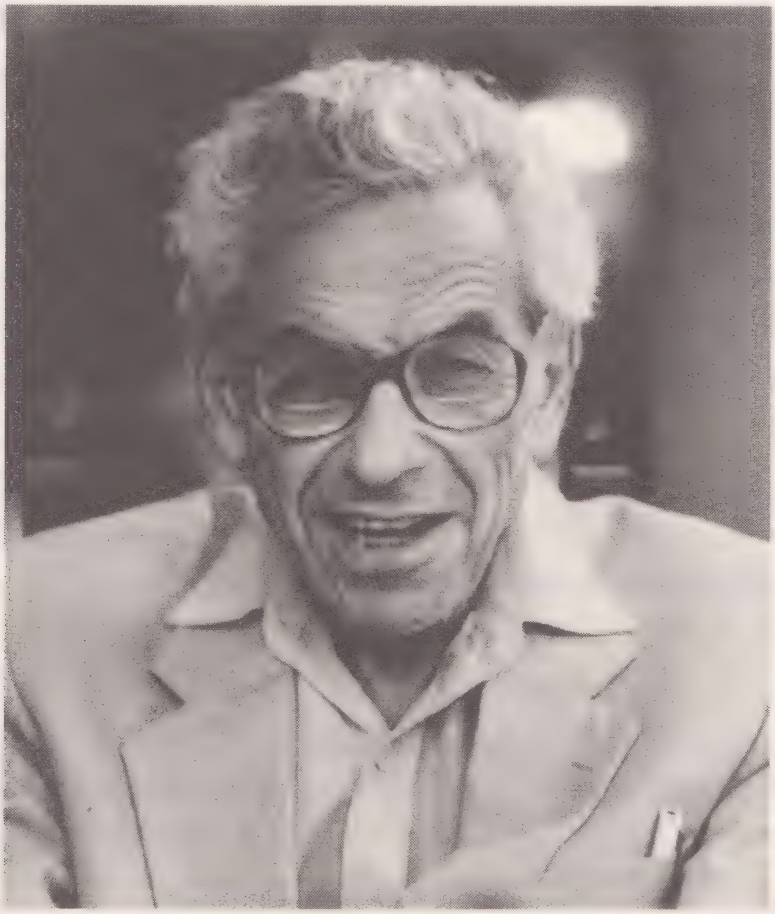
The image of Hungarian mathematical genius Paul Erdös (1913–1996) has grown with time, partly due to the extravagance of his personality and also because of his genuine contribution to the theory of numbers, on which he focused his formidable intellectual talents. And formidable he must have been, as he remembered having discovered negative numbers all on his own at just 4 years old.

Recounting anecdotes about Erdös is inevitable, as everything about his character and behaviour lend themselves to their inclusion. For example, he, like Hardy, thought that God was his natural enemy and that he was conspiring with reality to hide the most beautiful theorems from Erdös, who had to make great efforts to get them from him. He said that the gems of this dark knowledge were included in an imaginary book of intellectual marvels; so, when Erdös managed to demonstrate something particularly beautiful he would exclaim: "That must be in the book!"

Erdös is now a legend, and some structures linked to his character have entered the Valhalla of science. They include the Erdös number, initially a humorous creation which is today studied in graph theory. For any professional scientist X it is defined as the minimum number $E(X)$ such that they have co-authored at least one article with someone whose Erdös number is $E(X)-1$. As you will see, the definition is recursive; it culminates when the Erdös number 1 is defined. A scientist with an Erdös number of 1 is the co-author of an article with Erdös himself. Of course, anyone who has not written an article with Erdös but has done so with the fortunate owner of an Erdös number of 1, has an Erdös number of 2. Anyone who has written an article in collaboration with an X with $E(X)=2$, has an Erdös number of 3, and so on. Anybody outside this chain of co-authorship has an Erdös number of infinity. It is a deliciously mathematical way of classifying mathematicians.

The set of owners of an Erdös number of 1 has 511 members. Among those on the list is famous baseball player Hank Aaron due to the fact that, at the suggestion of Carl Pomerance (b. 1944), another mathematician, Erdös signed a baseball for him when he attended the same honorary doctorate ceremony. It has been calculated that nine-tenths of current scientists have an Erdös number of less than or equal to 8. So far the highest known Erdös number is 15. Calculating the definitive minimum upper level is a problem for you to try to solve.

It is worth noting that Richard Dedekind (1831-1916) is the oldest mathematician belonging to the illustrious brotherhood, E(Dedekind) = 7.



"My mind is open", Paul Erdös would say to his friends when he knocked on their door to stay in their houses for a while, armed with just his suitcase, one change of clothes, his grey matter and an excellent disposition for jointly solving convoluted numerical problems. He would then follow it with another famous rhyming phrase: "Another roof, another proof".

A little help from Erdös

Erdös' capacity to get bored if mathematics was not being done in his surroundings was, apparently, infinite. He was once invited to dinner and, when he realised that he was actually there for dinner and not to talk about mathematics, his nose hit his plate and he fell asleep. The reverse is explained by Polish-American mathematician Mark Kac (1914-1984), one of whose seminars was about a subject which Erdös was not particularly interested in. However, at one point Kac admitted to having run aground, as he could not solve a problem related to the divisors of a number. At that point Erdös' sense of smell was aroused, literally, like a cat smelling blood.

Erdös delved into the numerical problem like a man possessed while the lecture continued. Before Kac finished speaking, Erdös lifted his head triumphantly – he had solved the problem.

Mr Smith's numbers

It all started because of Albert Wilansky, a numerologist who invented a category of numbers from his brother-in-law's telephone number, or at least that is what we are told in the theory of numbers. Let's take it one step at a time: Wilansky's brother-in-law, one Harold Smith, had the telephone number 4937775. The sum of the numbers is 42:

$$4+9+3+7+7+7+5=42.$$

Wilansky factorised the telephone number:

$$4{,}937{,}775=3\cdot 5^2\cdot 65{,}837,$$

and wrote it without exponents, studiously repeating the prime factors:

$$4{,}937{,}775=3\cdot 5\cdot 5\cdot 65{,}837.$$

And, surprise, surprise, he also got 42 by adding all the figures:

$$3+5+5+6+5+8+3+7=42.$$

If it were anyone other than Wilansky it would have gone unnoticed, but it flicked a switch in his mind. Smith (the most common English surname) numbers had just been born. A Smith number (we define it in base 10, but any base is valid) is any prime number in which, when factorised and written as we saw above, the sum of its digits and the sum of the factors are the same. It may seem a somewhat elaborate definition, but actually the study of Smith numbers has been fruitful and there are hundreds of them, and thousands of numerologists harvest them and regard them with great affection. We now know that there are infinite Smith numbers, that among them there are infinite palindromes and we even know of some intriguing examples, such as the enormous

$$9 \cdot R_{1031}(10^{4594} + 3 \cdot 10^{2297} + 1)^{1476} \cdot 10^{3913210},$$

where R_{1031} (R comes from *repunit*, abbreviation of 'repeated unit') represents the integer formed by 1,031 repeated number 1s or, if you prefer,

$$R_{1031} = \frac{10^{1031} - 1}{9},$$

and which, in 2010, was the largest known Smith number. But it was not the most intriguing, as the so called 'number of the beast' which appears in St. John's Apocalypse, 666, is a Smith number. Let's take a look at it:

$$6 + 6 + 6 = 18,$$

and, on the other hand,

$$666 = 2 \cdot 3 \cdot 3 \cdot 37;$$
$$2 + 3 + 3 + 3 + 7 = 18.$$

Kabbalists and other lovers of the dark, be afraid. It is a shame that the Smith numbers have such a trivial name and that their creation is owed to a mundane telephone number.

The fly

Many anecdotes are attributed to US physicist and mathematician of Hungarian origins John von Neumann (1903-1957), perhaps because of his exceptional personality. One of the most quoted in the anthologies refers to both his calculating abilities and his curious habit of acting unlike other mortals. We are going to simplify the information in order to make the tale more accessible. A now classic problem is that of the trains and the fly. Suppose that two trains, A and B, leave two ends of the same line, one from point A and the other from B. Let's suppose that the distance between A and B is 100 km and that the trains travel at 50 km/h each. At the moment of departure, a fly which was resting on the nose of train A moves towards B at 75 km/h; it leaves train A behind and meets train B. Then its route reverses: it leaves the front of train B and flies towards that of train A. When it reaches it, it goes towards the front of train B again, and so on. Both trains end up meeting and the fly meets its maker. The question is, what is the distance travelled by the

fly and its infinite route that yo-yos back and forth? The distance travelled by the fly is, just as a good degree student would show after a laborious calculation, the sum of the infinite geometric series

$$d = 60 + 12 + \frac{12}{5} + \frac{12}{25} + \ldots,$$

which is equal to 1/5 and gives us $d = 75$ km for the dipteran's flight.

A simple soul with a sharp brain would arrive at this answer as follows. Trains A and B will meet at kilometre 50, the middle of the full track, and, due to their velocity, it will have taken them one hour. So the fly will have been flying for one hour, and as it moves at 75 km/h it will have travelled 75 km. It is obvious, but few people hit upon such a simple proof.

Now let's go back to Von Neumann and a colleague of his who proposed the fly puzzle to him. Von Neumann immediately responded: "75 km". His colleague looked a little disheartened: "Wow, there's no catching you out. You got it straight away because you're so intelligent, but most people do it by adding a series". And Von Neumann answered in surprise: "And what do you think I did?". The genius of geniuses had not put a single second into simple lateral thinking; he had limited himself to working out and calculating the series instantly. Quick and simple... if you are Von Neumann.

Fermat's room

Now and again a famous or simple mathematical problem has been taken to the big screen, such as in *Little Man Tate* (1991), *Cube* (1997), *Moebius* (1996), *Pi (Faith in Chaos)* (1998), *Enigma* (2001) and many other films. However, there is one where everything revolves around mathematics. It is *La Habitación de Fermat* [Fermat's Room] (2007), co-directed by Luis Piedrahita and Rodrigo Sopeña and interpreted by an excellent cast of actors. Alejo Sauras' character, who assumes the role of a young professional who responds to the code name Galois (a wink to the viewer), is of special importance as he has come up with none other than the demonstration of the Goldbach conjecture. Unfortunately, he no longer has it because it was stolen, as he informs us at the beginning of the film.

The film's plot is very complicated and has many twists, of which one of the more original ones is the fact that the main characters are enclosed in a room whose walls are moving inwards, threatening to crush them (although that idea seems to

have been taken from an Edgar Allan Poe tale). This truculent story is just a film. No one has come anywhere near the Goldbach hypothesis. In fact, the Galois in the film admits that his demonstration was false, but another of the characters (who answers to the appropriate alias of Hilbert and is played by Lluís Homar) does seem to have found actual proof. Unfortunately, Hilbert dies and the eventual proof ends up at the bottom of a river. So, in the end, the Goldbach conjecture ends up as it was and the desolate world of mathematics still awaits its charming prince.

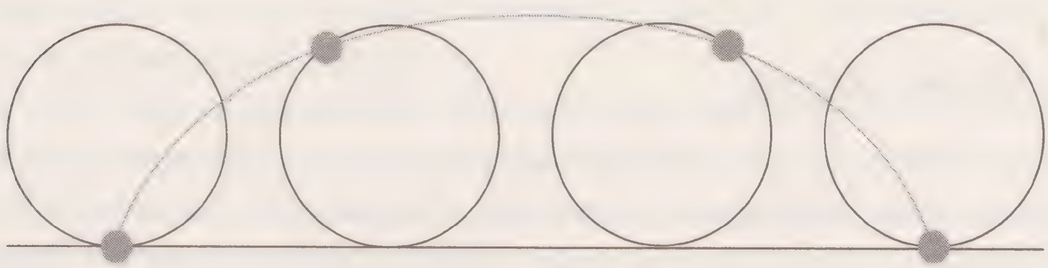
Chapter 2

Shapes

Geometry is the only science that it hath pleased God hitherto to bestow on mankind.
Thomas Hobbes

Cycloids instead of sheep

Counting sheep is the typical technique of insomniacs – the less the sleep, the more the sheep. Theologian and mathematician Blaise Pascal (1623-1662) came to see things differently. Towards the end of his life his thoughts were diverted almost exclusively to theology, leaving aside scientific matters, which had been his main occupation until then. But as well as suffering from irresistible religious compulsions, Pascal commented that he also suffered from insomnia, an annoying condition which he was unable to improve, even by counting sheep. It seems that the lack of sleep was accompanied by a continuous toothache, a condition which, at a time before pain killers, must have been excruciating. Perhaps through his other activities it occurred to him to think of geometric objects and their mysterious charms during his periods of unrest, such as the cycloid curve. In doing so he distracted his mind from the pain. Whether or not the cycloid had special healing powers, the fact is that the toothache that accompanied the insomnia and the insomnia itself disappeared.



The cycloid is generated mechanically; it is the path of a fixed point on a rolling circle.

Reflecting on this intriguing phenomenon, Pascal later found no other explanation than a religious one. It would seem that the Lord liked mathematics

more than anything else, including zoology. Pascal even created a kind of prize for anyone who could provide him with interesting results on the cycloid, and named the well-known specialist Gilles Personne de Roberval (1602-1675) as a member of the panel.

There is a little more we can reveal about this fussy character who was also an excellent mathematician. He adored the cycloid, a curve which was at the centre of so many conflicts and disputes that many geometricians called it 'the Helen of Geometry', claiming a scientific parallel with Helen of Troy. It was Roberval who was the central figure in many of these disputes and for a very simple reason. The post of head of mathematics at the Collège de France was appointed every three years and was marked with a competition on a subject chosen by the outgoing chief. As you would expect, the head kept the results secret until the time of the final announcement – and he usually won the competition since he had the advantage. The problem was that if anyone discovered the secret theorem on their own and revealed it before the head, all hell would break lose. And Roberval held the position of head for no less than 40 years, enough time to make enemies and fight with everyone. Pascal was a good Frenchman, like Roberval, and when the latter fought with the Italian Torricelli over a conflict over precedence, Pascal took his side. It was not the right decision because, as has been shown subsequently, Torricelli correctly calculated the area under the curve and the method for drawing its tangents completely independently of the temperamental Roberval.

Let's finish off with Pascal. His father did not look kindly upon his love of mathematics until, when he was just nine years old, he took an interest in geometry and discovered without help from anyone and off his own back propositions that covered the content of Euclid's first 32 theorems. Only then did his father give in to the wishes of the boy.

The elusive polygon

The boy prodigy who was Carl Friedrich Gauss found out at the age of 19 which regular polygons could be constructed with ruler and compasses alone and which could not. At the time Gauss was unsure whether to dedicate himself to linguistics or mathematics, fields in which he showed amazing aptitude. When he managed to unravel the secrets of the polygons, he realised that his calling was geometry, so he made a decision: he chose the route of mathematics and he had no reason to regret it. He was the indisputable leader in the field for many years.

Gauss discovered the secret of the polygons through the following result: a regular polygon with n sides is constructible (that is to say, it can be constructed according to Greek tradition with only a ruler and a compasses) if it is of the type

$$n = 2^k p_1 p_2 \cdot \ldots \cdot p_m \text{ where } k \geq 0,$$

and where p_i is either 1 or one of the other prime Fermat numbers. The only thing in the theorem left to explain is what a Fermat number is. A number F_p is called a Fermat number when it is of the type

$$F_p = 2^{2^p} + 1.$$

Fermat numbers can be prime or composite.

$$\begin{aligned}
F_0 &= 2^{2^0} + 1 = 3 \\
F_1 &= 2^{2^1} + 1 = 5 \\
F_2 &= 2^{2^2} + 1 = 17 \\
F_3 &= 2^{2^3} + 1 = 257 \\
F_4 &= 2^{2^4} + 1 = 65{,}537 \\
F_5 &= 2^{2^5} + 1 = 4{,}294{,}967{,}297 = 641 \cdot 6{,}700{,}417.
\end{aligned}$$

From this point nobody has been able to find a Fermat prime. F_6 was factorised in 1880 by Frenchman Fortuné Landry, and of the following F_p F_{11} has already been factorised, but as we have said, no more primes have been found; perhaps there are none to find.

The theorem means that regular polygons with n sides are constructible where n = 3, 4, 5, 6, 8, 10, 12, 15, 16, 17, 24… up to 65,537, which is the same as F_4. We will stop here to mention that there is an apparent manual for the construction of the corresponding regular polygon.

In 1894, German geometrician Johann Gustav Hermes (1846–1912) finished the almost inconceivable construction procedure, which fills more than 200 pages. As he could not get it published, he submitted the manuscript to the University of Göttingen and there it remains, as perhaps it always will, as there are some who doubt that the construction is correct. What a let down if, after so much effort (British geometrician H.S.M. Coxeter calculated that the work must have taken the author about ten years), an error is discovered. Of course, it is unlikely that anyone will dedicate another ten years to checking it.

A true gentleman

Gaspard Monge (1746-1818) was not a true gentleman; he was born and raised among itinerant traders. His life revolved around Napoleon Bonaparte (1769-1821), whom he followed on his youthful expedition to Egypt and whom he subsequently never abandoned. After his death the Bourbon monarchy prohibited students of l'École Polytechnique from attending his burial. His remains are currently buried in the Pantheon. Monge was the creator of descriptive geometry and one of history's most important geometricians in his field and in that of differential geometry. An entire book could be written on the events of his tumultuous life, but we are going to confine ourselves to an episode that presents his everyday life.

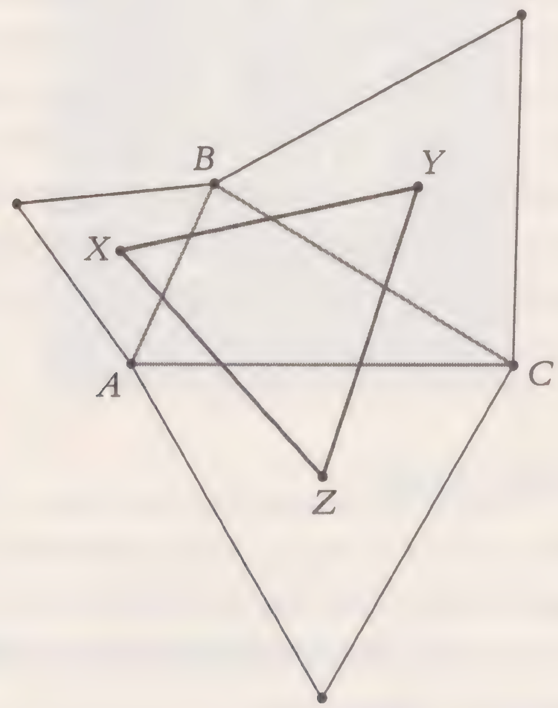
In his youth he frequented a bar and happened to overhear a patron (of noble birth) badmouthing a widow named Horbon, who had rejected him and whom, on a wave of bitterness, he accused of all the world's evils. The gallant Monge could not tolerate such offence aimed at an absent lady, and he confronted the nobleman in question. Confront him is not all he did, as it seems that fists flew and then there was a formal challenge to a duel by Monge, which never came to be. After a while, Monge had the fortune to be introduced to an adorable, young and beautiful widow, whom he ended up falling for. The lady did not want to marry until the matters of her deceased husband were completely dealt with. You will already have guessed that the widow in question was none other than M. Horbon. They married in 1778 and, as they say in the fairy tales, they lived happily ever after, as Napoleon bestowed Monge with the title of Count of Pelusium. In fact commentators at the time used the Monge nuptials as an example of the model French couple.

Napoleon's theorem

Perhaps the most inoffensive of all the things that Napoleon Bonaparte did in the free time he found between enacting laws, building empires and planning battles was proving theorems. Napoleon was an amateur mathematician, who never reached a professional standard because he was busy starting wars, as we all know. He felt very comfortable surrounded by mathematical luminaries – Fourier, Monge, Laplace and many others formed part of his entourage – to the desperation of his generals, people cut from a different cloth and whose most immediate interests lay in the illumination of the enemy rather than new geometric constructions with a ruler and compasses. It is said that his military staff slept during meetings in which the

mighty Corsican talked with his intellectual attendants. The fact is that Napoleon went as far as subjecting his marshals to geometry lessons, given by the well-known geometrician Lorenzo Mascheroni (1750–1800). Although it is not because of this that Napoleon deserves a place in the pantheon of the immortal mathematicians.

The so-called Napoleon theorem, attributed to the emperor, establishes that if equilateral triangles are constructed on the faces of any triangle, the centres of these new triangles will also determine an equilateral triangle. A drawing will help with the understanding of this attractive result, which is considered an elemental theorem in Euclidean geometry:



The statement may be from the Napoleonic period, but its proof, according to experts, is not Napoleon's. The wording of the theorem can be traced through more than a century, 'proven' by someone different each time. The oldest proof found is that of Dr. W. Rutherford, and it is dated 1825, four years after the death of Napoleon on Saint Helena, and unless he is granted the ability to win battles after his death, much like El Cid, he did not triumph in that of his theorem. Furthermore, Rutherford published his solution in an annual almanac aimed at 'the fairer sex', *The Ladies' Diary*.

The Napoleonic theorem was the subject of generalisations in the last century. Adriano Barlotti proved it, not for equilateral triangles, but with regular *n*-sided polygons.

On safari

Today the lands of the Congo do not enjoy great prestige. It is one of the most undeveloped areas on the planet and one of the bloodiest (and least noticed) warzones in the world. It is also the homeland of the Bakuba people, a tribe of geometricians, where the design of symmetrical stripes is of special importance. The designs can be seen on masks, the cloths which cover their heads, on the royal drum and even on the dynastic statues.

When it was agreed to include the Bakuba in a list of 'civilised people', a long-standing tradition was followed and a gift was made to the king. In the case of the Bakuba people this was a motorbike, a miraculous vehicle to a Congolese tribe during the early 20th century. Tradition demands that the response to the gift be one of amazement and submission. However, the response was unexpected and it seems that geometry got the better of them. Nobody paid the least bit of attention to the motorbike, but they loved the tyre prints. The prints left by the tyres were symmetrical and had an interesting design, such that they were copied, and the monarch even put his name to the design. How is the value of something really measured? Is there a universal criterion?

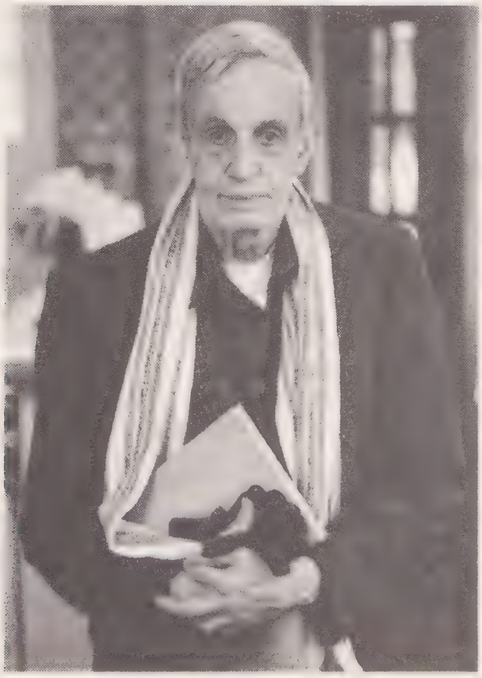
A marvellous chaotic mind

Few mathematicians have made it to the big screen, and of those who have, perhaps the most famous is John Forbes Nash, the protagonist of *A Beautiful Mind,* who became mentally ill in the midst of his mathematical peak only to be cured many years later, in time to collect a Nobel prize.

John Nash (b. 1928) was not only a Nobel prize winner whose life was told on film, but also an excellent mathematician, a genuine prime specimen. He is the proud owner of an Erdös number of 4. In books on algebraic geometry, he always appears as the author of a theorem that carries his name, and which states that every Riemannian manifold can be isometrically embedded into some Euclidean space. This important theorem was published in the *Annals of Mathematics* magazine in 1952, under the title 'Real Algebraic Manifolds'. Legend would have us believe that the real scribe of the article was someone else – the editor of the magazine – because due to his illness, Nash's ideas were by then so confused that when the original was sent it was like an inextricable jungle and was almost impossible to follow.

The article was rewritten and published and was very well received. The only thing is that maybe there is no truth in this, as Nash only began to show symptoms

of paranoia six years later in 1958, and the dates do not quite match. We know a lot about Nash and it is all very film-worthy, but the story about the article is hard to believe.

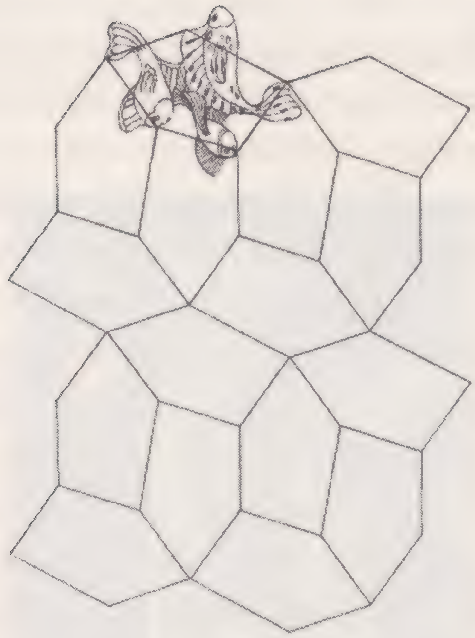


John Forbes Nash (source: Peter Badge).

Housewife by day, geometrician by night

It is not considered normal for a simple housewife, in the time between cooking macaroni and sewing, to show the world that her grey matter works on the same level as Hercule Poirot. This is exactly what Marjorie Rice (b. 1923) did, a busy mother of a Californian family, a lady who had always really liked mathematics and puzzles, who began by helping her children with their maths homework when she got married and ended up astonishing the experts.

A copy of *Scientific American* fell into Marjorie's hands in which the famous columnist Martin Gardner (1914-2010) explained the results of convex pentagonal tiling on a plane by R.B. Kershner. Marjorie, working in her free time and taking occasional notes in the privacy of her kitchen, found four convex pentagonal tilings, which were completely new, in the following two years. The following drawing gives an idea of Marjorie's results:



A fragment of Marjorie's pentagonal tiling. In each pentagon the shape of the one above is repeated. It is called 'fishes' because the plane is completely covered by fishes, giving it the appearance of one of Escher's famous engravings.

It is far from the only time an 'amateur' mathematician has come to the fore, which just goes to show that, when it comes to thinking, titles often count for little.

The strange case of the genius who decided to stop being so

A lot of ink has already been spent on the case of Alexander Grothendieck (b. 1928), a stateless mathematician of German origin who is considered by many to be French. If we had to classify this true genius, we would include him among the geometricians, as his contributions to the field of algebraic geometry have been fundamental, although they are very, very difficult to explain. In any case, the mathematical world in which he operated was so abstract that it is difficult to pin it down to one particular place. And we are talking in the past tense despite the fact that he is alive because Grothendieck retired from public view in 1988, and all we know is that he has given up mathematics and that he lives in the south of France, isolated from his ex-colleagues and, it seems, from the world.

His story is quite dramatic, and it includes Jewish origins, a solitary childhood (his parents abandoned him to fight for the Republican cause in the Spanish Civil War), a father who died in Auschwitz, some controversial marriages, fanatic pacifism, extreme political radicalism and, on the positive side, a privileged mind that allowed him,

time after time, to invent hugely advanced concepts and theories. More than a dozen modern mathematical concepts, some of them with colourful names (Grothendieck schemes, topology and spaces, children's drawings, crystalline cohomology, etc.) carry his name or are owed to him.

In 1966 Grothendieck was awarded the Fields medal and he turned it down. Nor did he accept, in 1988, the distinguished Crafoord Prize, the Nobel for disciplines that do not have a Nobel. In his golden years he worked in the Institut des Hautes Études Scientifiques, near Paris. He stopped doing so when he was informed that part of the funding came from sources linked to military organisations. In 1978 one of his doctorate students, Pierre Deligne (b. 1944), received a Fields medal of his own, something that was very unusual.

His torrential and confusing memoirs were published along with various documents, all of which are enormously long. Contact with him has been rare, and only by correspondence. More than 20 years have passed without any scientific contributions from him, and there is no hope of news of him, unless it is of his definitive disappearance from among the living. It is a shame because that is some mind Grothendieck has!

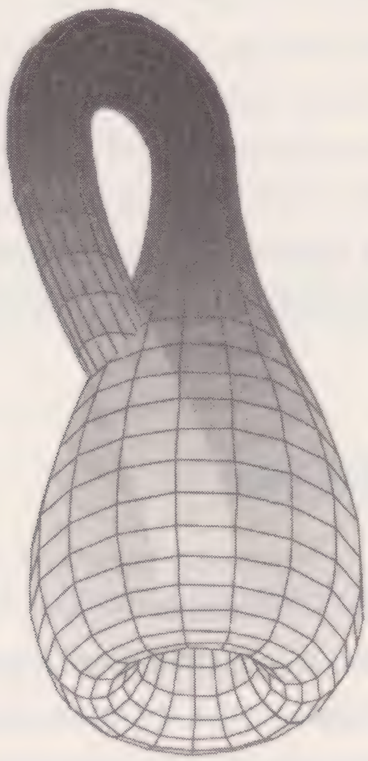
Tony Blair and the rhinoceros

While there are rulers who are amateur mathematicians, like Napoleon and Éamon deValera, others are not so keen. This was not the case, for example, for US president James Garfield (1831-1881), who found a new demonstration of Pythagoras' theorem during a particularly slow legislative session. However, there is one ex-leader who is particularly non-mathematical, if we are to believe the story told by his teacher at The Chorister School, Durham, where the young Tony Blair was a student. Blair's answer to a particular question involving triangles and rectangles was a sentence in which the word 'rhinoceros' mysteriously appeared. When duly interrogated, Blair confessed, displaying an embryonic skill for creating linguistic 'wriggle room'. The word was strange, of course, but his reasons were, in a way, charitable: "I would have written 'hippopotamus', but as I was sure that it was not the exact word, I did not want to upset you and I put the first, more or less similar, thing that came to mind". What Blair should have written was 'hypotenuse'.

The vocal confusion between the terms 'hippopotamus' and 'hypotenuse' is a classic in the world of apocryphal mathematical anecdotes. We have cited this here because it involves a relevant public figure and because it is well documented.

A bottle without an outside or inside

As Captain Haddock, Tintin's companion, would say, all the bottles in our world have an outside or an inside. They are empty or they contain something. Or nearly all of them, because there are mathematical bottles (which were unknown and useless to Haddock) with very strange properties. In 1882 the German mathematician Felix Klein (1949-1925) designed a bottle which, as can be seen, has no interior or exterior, no inside or outside; you cannot drink from it, no matter how hard you try.



Filling it or emptying it makes no sense; the reader can try with a stretch of the imagination. It is an object which, unfortunately, penetrates itself in our three-dimensional universe. In the fourth dimension it would not penetrate itself, and it would be a perfectly conceivable object. From a strictly geometric point of view, the Klein bottle is a closed, non-orientable surface with no edges; it is studied in topology, together with its flat sister, the Möbius strip.

The interesting thing about this popular geometric monstrosity lies in its name, which is a tribute to the phenomenon of the confusion of languages; it was originally called, in German, *Kleinsche fläche*, or, 'Klein surface', which correctly described the object. If anyone should want to draw such a surface (only

a computer program and a printer are needed) it can be done with the Cartesian equation:

$$\left(x^2+y^2+z^2+2y-1\right)\left[\left(x^2+y^2+z^2-2y-1\right)^2-8z^2\right]+16xz\left(x^2+y^2+z^2-2y-1\right)=0.$$

However, even mathematicians do not use the correct term, and *Kleinsche fläche* was transcribed as *Kleinsche flasche*, which means 'Klein bottle'. The mistake describes Klein's surface so well that the name has been imposed on the international scientific world, even in German! A great example of cybernetic feedback.

A flourishing business has been created around Klein's discovery, in which the highly requested items are woollen hats for cold weather in the shape of the Klein surface and Klein wine carafes which are almost, but not completely, immersions of the Klein bottle into the third dimension. They actually do pour wine – just about. In the fourth dimension that would not be possible (they cannot be filled) but in the third dimension they are so impractical as to be practically useless.

Chapter 3

Calculus

What are these fluxions? The velocities of evanescent increments? And what are these same evanescent increments? They are neither finite quantities nor quantities infinitely small, nor yet nothing. May we not call them the 'ghosts of departed quantities'?

Bishop George Berkeley (1685-1753)

This passage is from the booklet *The Analyst* (1734), a magnificent intellectual exercise by an Anglican bishop, dedicated "to an infidel mathematician" (this appears to have referred to Edmond Halley (1656-1742), he of the comet, a notorious non-believer). The bishop's booklet attacks the recently developed Newtonian calculus, which was so adored by Halley and by the scientific world in general, reproaching them, not without some justification, saying that if they did not believe in God because the writings of the sacred texts were incomprehensible, then they should not believe in the quasi-mystical trickery of calculus.

Years passed (or rather, centuries) and the edifice of infinitesimal calculus recovered its credibility, based on rigour and more precise definitions which were less intuitive. This does not mean that Berkeley, an excellent empiricist philosopher (the famous US university campus carries his name), should be ignored; instead he should be given respect for his sound and well-grounded criticism.

The weapons thought up by Newton and Leibniz opened the way to and generated many situations that we would call anecdotal; here we have gathered a few of them.

Bishop George Berkeley painted by John Smybert.

Conjectures, theorems and Newton

Evidently, a conjecture is not the same thing as a theorem. When a conjecture is proven it acquires the title of a theorem, and this is something to which little attention has been paid throughout the years.

Let's take Johannes Kepler (1571-1630), for example. We have all waxed lyrical about Kepler's laws; after all, they are empirical deductions based on data tables by Tycho Brahe (1546-1601). They are also undoubtedly quite brilliant and were revealed to the scientific world and gradually accepted by it, but they had no mathematical proof whatsoever. Kepler's three laws appeared to govern the movement of the heavens, something which he convinced the astronomers of his time of little by little. But, from the vantage point of current knowledge, all we can say is that they were three brilliant conjectures, not that they were three true mathematical statements or theorems.

Along came Isaac Newton (1643-1727) and, more than half a century later, he shed light on the matter. It was Newton who, by applying the basic laws of infinitesimal and integral calculus to mechanics, deduced Kepler's three laws from the fundamental hypothesis – the inverse squared law, which establishes that two bodies are attracted with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance which separates them. And, talking of Newton, a luminary of thinking who is always said to have been a bit cold and aloof, here I have a story about him that is clearly apocryphal, but which also humanises his character a little. Newton had a dog called Diamond (this is true), to whom he jokingly attributed mathematical abilities. In one particular interview with Wallis, he joked: "Today Diamond has proven two theorems before lunch." Wallis played along: "Your dog must be a genius." Newton's answer: "Oh I wouldn't go that far. The first theorem had an error and the second had a pathological exception."[1]

Whoever does it, pays for it

One day in 1684, Edmond Halley, Sir Christopher Wren (1632-1723), the architect who designed St Paul's Cathedral in London, and Robert Hooke (1635-1703), he who, among other things, coined the word 'cell', were talking in a coffee shop after a meeting at the Royal Society. They were discussing what shape of curve would be

1. In mathematics, a 'pathological exception' is one which is so aberrant that it alone lowers the scientific (and aesthetic) value of a general principle which, otherwise, holds true.

adopted by a planet attracted by the Sun with a force that was inversely proportional to the square of its distance to the centre of attraction. Wren even offered a monetary reward for anyone who demonstrated it. Hooke said that it was an ellipse, but he never actually proved it. The meeting was adjourned.

Shortly after, Halley visited his friend Newton and, in the middle of the conversation, he asked him what shape the curve would be. "Elliptical" Newton immediately and confidently replied. "How can you be so sure?" Halley said. "Because I have calculated it," came the answer. Halley must have been counting his lucky stars, as Newton was not the type to make such statements for free. The demonstration did not appear in the papers that Newton had to hand because it was a while previously that he did those calculations! To cut a long story short, he did them again. Encouraged by Halley, Newton wrote what he had deduced from the law of the inverse squares, one thing led to another, and in 18 months the *Philosophiæ naturalis principia mathematica*, popularly known as *Principia* was born, a piece of work that is fundamental to our comprehension of the Universe. Newton contributed gravity, the law of the inverse squares, ellipses and the grounds of infinitesimal calculus. And he squeezed all this, and much more, into three thick volumes. After a few months there were scientists who practically wept at such magnificence and wisdom. All of this was provided by Newton, who did not have the money to pay for the book and who had written it at Halley's suggestion. So, who paid for Newton's book and ended up completely out of pocket? Halley, of course.

Mathematician and astronomer Edmond Halley was the first to calculate the orbit of a comet, the one which now bears his name.

Halley is the character who is known to the greater public as the person who calculated the orbit of the comet that bears his name. It is a recurring comet that appears every 75-76 years, with an apparent magnitude of 28.2 (when it passed in 2003), and is visible to the naked eye. Halley saw it in 1682 and, using data from the observation, Newtonian mechanics and his own daring intuition, he concluded that the comet observed in 1531 by Petrus Apianus and in 1607 by Kepler were the same one he had seen with his own eyes.

Halley made his first observation in 1682, therefore, if he was right, the next appearance of the comet would not happen until 1758. That was a long time considering that Halley was not a young man and, indeed, when the comet came back right on time and true to Halley's calculations, Halley had died 16 years previously. It is uncertain what would have been the greater achievement, living to 102 or being able to confirm his prediction.

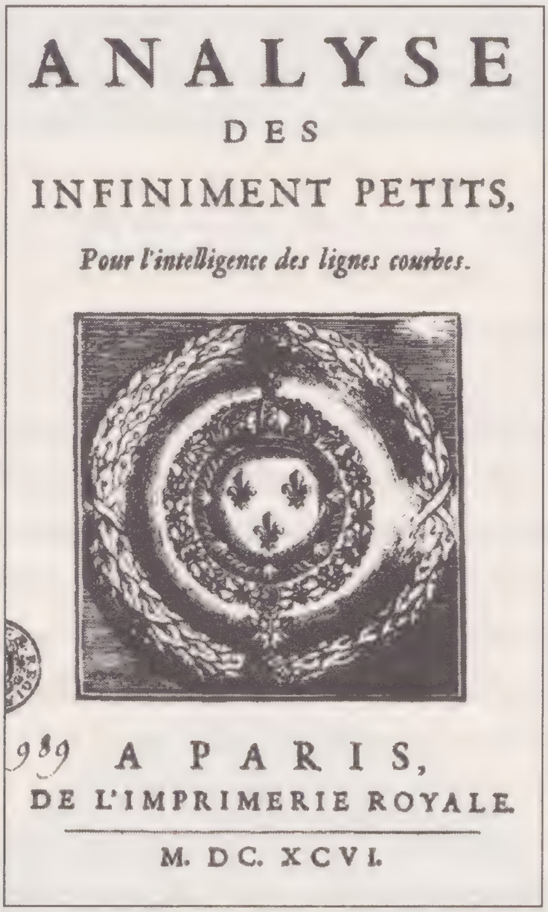
The ostentatious marquis

The following story highlights the fact that, often, where there is money there is ostentation. It all starts with the Bernoullis, a Swiss family who we will later dedicate a few lines to, and with the Marquis de l'Hôpital (the marquis wrote his surname as 'Hospital'). His complete name was Guillaume François Antoine de l'Hospital, Marquis of Sainte-Mesme and Conde d'Entremont (1661-1704), and he had an apparently embarrassing incident with the aforementioned family of mathematicians.

The marquis was an excellent mathematician, and in that regard we cannot criticise him. As he had a lot of money, he wanted to make it available for mathematics, and perhaps to benefit his good name, and so he paid the brilliant Johann Bernoulli for the rights (as the historian William Dunham derisively describes it) to his discoveries. Nowadays this sounds horrific, but to Johann and the marquis it cannot have seemed so bad. Johann Bernoulli's research appeared in 1696 under the title *Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes* – the only thing in the book, according to Dunham, that was the marquis' own work. The text was very good and well received.

In 1704, with l'Hôpital having passed away, Bernoulli revealed the truth behind what had happened. But it should be said that his theory, which was correct, was not believed by everyone. Bernoulli's tricks were fairly well known and Johann's reputation, fairly dubious.

In 1921 papers were found that indicated that Johann really was responsible for most of the discoveries, but even so it is not clear whether the marquis sought undue credit: firstly l'Hôpital was actually a qualified mathematician; secondly, the book was published anonymously, without a mention of authorship and thirdly, the prologue mentions Johann at length and thanks him for his contribution. Perhaps the marquis just wanted all mathematical knowledge to be available to everyone.


ANALYSE
DES
INFINIMENT PETITS,
Pour l'intelligence des lignes courbes.

A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DC. XCVI.

Cover of the first edition of the Marquis de l'Hôpital's most famous text.

And while we are on the subject, let's dedicate a few lines to the Bernoulli family. The first Bernoullis were Jakob (1654-1705) and Johann (1667-1748); later the Bernoulli dynasty evolved with the addition of Johann's son, Daniel Bernoulli (1700-1782) and their nephew, Nicolaus Bernoulli (1687-1759), also eminent mathematicians. But it did not stop there. By 1807 there were no less than nine Bernoullis, all of them notable scientists. The Bernoulli family is the scientific equivalent to the Bach family in music. Although perhaps the difference lies in some bad relationships between some members of the family. Some of their disputes have

become legendary, such as when Johann fought with his own son Daniel, stealing a part of his results on hydrodynamics. The things jealousy makes us do...

The miller's integral

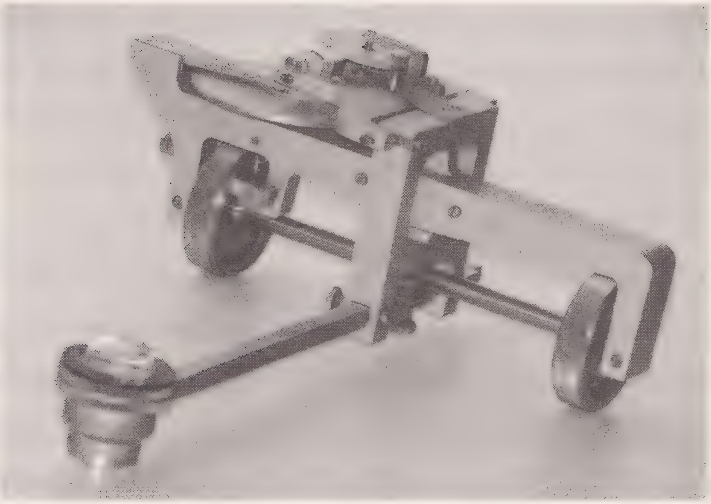
'Amateur' mathematicians have always fascinated people. Amateurs rarely follow a conventional path when acquiring their surprising knowledge, and, in many cases, they are unusually skilled, such as the Pole Stefan Banach (1892-1945) or the Indian Srinivasa Ramanujan, to give two examples of uneducated minds in the mathematical world who have reached the lofty pinnacle of intellect. The king of amateurs may be Pierre de Fermat (1601-1665), the lawyer who read arithmetic books, the margins of which were unfortunately too narrow to contain all of his proofs.

A good example of a self-taught scientist is George Green (1793-1841), who acquired his mathematical knowledge in academic solitude. He also had an unusual characteristic. In his native England at the time it was frowned upon to prefer the ideas behind the infinitesimal calculus of Leibniz – from continental Europe – to those of the home-grown and almost worshipped Newton. But this is exactly what we can deduce from the work of Green, given his undoubted preference for the transparent Leibniz notation to the murkier Newtonian version.

Despite this, what is truly surprising is Green's background: he was a miller. The son of a wealthy baker, Green did not dare enrol at Cambridge until he was 40, even though he was encouraged to. His work gave us what is today known as the Green theorem (the Russian Mijaíl Ostrogradski (1801-1861) presented it independently), with repercussions that led to the differential and integral calculus we have today.

$$\oint_C (L\,dx + M\,dy) = \int\int_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dx\,dy.$$

His work has even led to substantial advances in quantum mechanics, a field that was unimaginable to him. From Leplace's *Mécanique céleste*, Green managed to deduce a comprehensive mathematical theory for electricity. We should not omit that during his latter years Green was also accused of drinking too much. This miller (the mill is now a museum) must have been a pretty ordinary guy.



One of the real-life applications of the Green theorem is the planimeter, an instrument that measures the area enclosed by an irregular surface by tracing its perimeter.

Soapy surfaces

Belgian physicist Joseph Plateau (1801–1883) was a great experimenter who contributed hoards of information to science on how the retina works and the persistence of vision. We owe the phenakistoscope to Plateau. Today we are so used to the cinema that Plateau's inventions, and their derivatives, have been relegated to form part of recreational physics, but they were the fundamentals of cinema itself.

The phenakistoscope was the first known application of the persistence of images in the eye; the figures of the phenakistiscope seem to move as the disc rotates.

All of this would have little to do with mathematics if it were not for the fact that Plateau carried out experiments, almost casually, with oil-based substances, producing observations about surface tension and soapy surfaces. If a curved structure that represents the outline of a surface (for example, a wire that has been bent and curved to represent its border or limit) is submerged into a soapy liquid, a film of soap spreads across the shape with a minimum surface area. In other words, the soapy surface formed by submerging the wire is the minimum surface area outlined by the wire. Here lies the mathematics of the matter. Finding a minimum surface area with the instruments of calculus (calculus of variations, first-order partial derivatives, etc.) is very complicated, sometimes impossible. Finding a physical solution just requires water and soap. And that is the considerable contribution of Plateau.

The minimum surface area between two rings turns out to be a catenoid, not a straight cylinder, as demonstrated by this experiment with soapy films.

What is truly anecdotal is the story of Plateau's life. Many have called him a 'martyr of science'. During the course of one of his experiments in 1829 he observed the Sun with the naked eye for 25 seconds and became blind. The experience was, from any point of view, completely stupid, and so in legends Plateau is painted as someone who put everything into science, even his sight.

Posterity, which is more realistic, has established that although Plateau was blinded, his blindness was temporary, and after a while he regained his sight.

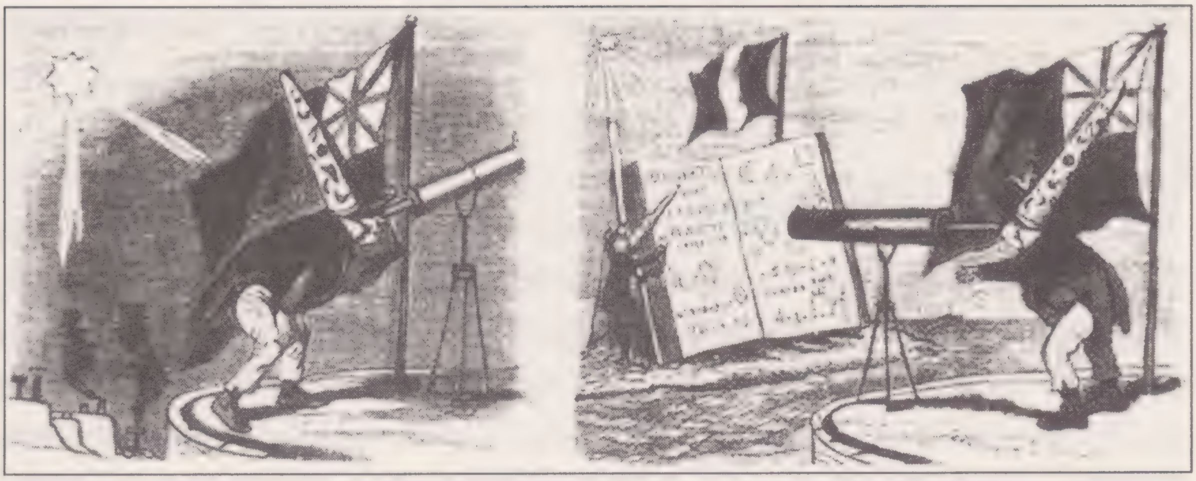
Years later, in 1843, he began to go blind again (for unknown reasons this time), and he died many years later, never having stopped his investigations after losing his sight.

The discovery of Neptune

The planet Neptune was discovered in 1846, and its discovery constituted a truly exceptional triumph for calculus. It was a discovery that we could say happened the other way round to the normal course of such events. It began with disturbances in Uranus' movement. The planet was not in precisely the expected places when observed, which may have indicated the existence of an unidentified object the gravity of which was disrupting its motion. The complete story of Neptune played the English and French scientific communities off against one another. The battle spread beyond their borders too, with respectable scholars up in arms and professional magazines publishing inflammatory articles and declarations. The discovery was attributed to Urbain de Le Verrier (1811-1877), director of the observatory at Paris, and the young English astronomer John Couch Adams (1819-1892), a man who, with time, would become a lot more famous.

Initially it was supposed that Le Verrier was first, and even that Adams' claim was somewhat suspicious, but historiography later proved that the Englishman was actually first by the skin of his teeth. In fact, Le Verrier's and Adams' discoveries were independent and neither of them was aware of the other's work. But it is true that it was work at an impressive mathematical level involving enormous, extremely long and complicated calculations. There was already a precedent for bloodless war between the insular Britain and the continent due to the Newton–Leibniz dispute over the discovery of infinitesimal calculus, such that the Le Verrier–Adams controversy was nothing more than a rerun of the previous one. There was no reason to argue in either case, but of course it is much more exciting to take potshots at one another than it is to remain level headed.

Le Verrier's intentions add an element of fun to the proceedings. Through the mouth of his representative François Arago (1786-1853), he sought to name the new planet not Neptune, but with the name of its supposed French discoverer (his own), and this was announced from the French academy of sciences. Needless to say, the notion was universally rejected and, despite the insistence of French science, the mythological name of the Greek god Neptune was adopted internationally. Today we identify the planet Neptune with a stylised trident, its astronomic symbol.



French engraving from the period which shows the astronomer Adams (on the left) trying to spy on Le Verrier's book of discoveries.

Common sense and mathematics

Thousands of stories have been told about Thomas Alva Edison (1847-1931), most of them apocryphal. This one could be too, but it does figure in several reliable sources. The anecdote involves a mathematician from Princeton called Upton whom Edison hired to work in his laboratories. Let's not forget that Edison was mainly self taught, and did not formally study engineering, which is the branch of thinking where his technological contributions best fit. He could not carry out complicated calculations himself and he needed qualified professionals working for him. Once Edison had Upton find the volume of a pear-shaped light bulb. Upton, armed with paper and the mathematical instruments of integral calculus, got to work. The days passed but Upton's computational orgy went on. Edison became impatient and it was then that he demonstrated his practical genius in all its glory: he took the light-bulb, filled it with water and gave it back to Upton: "Measure the volume of the water and there you have it." And he did have it, as clear as water.

The importance of being called Taylor

There was once a series expansion that saved a life. Physicist and mathematician Igor Tamm (1895-1971), who won the Nobel prize for physics in 1958, was not so well-known during the tempestuous times of the Russian revolution which began in 1917, when communists and non-communists fought for power. Driven by hunger, Igor left the relative safety of the city of Odessa and went to search

for food in the surrounding area. Unfortunately for him, one of the many armed groups operating at the time caught him and immediately accused him of being a camouflaged communist agitator. Shaking with fear and anticipating a summary execution, Tamm was taken to the group's leader. There he identified himself as best he could, explaining that he was a poor mathematician in search of food.

His interrogator began to idly play with his weapons and showed unmistakable signs of doubt. "OK," he said to him, "if what you say is true, you will live. Otherwise, you are a dead man. There is a simple way to find out: calculate for me right now the error when instead of an $f(x)$ function you take its Taylor series to the nth term." The question was not too difficult, as any student of analysis would testify, but it was completely unexpected from the lips of a delinquent outlaw, whatever his position in the group. Tamm, shaking, rushed to write the answer in the dust on the floor with his finger. The leader, adjusting his holster, took a look at Tamm's answer. "It is correct. You are not lying. You can go." They never saw him again.

A simple Pole, sir

The most notable characteristic of Mark Kac (1914–1984) was that he was a genius in the theory of probability. The second, lesser-known than the previous one, was that he was Polish. According to Kac himself, he once found himself in the presence of an untalented student. When he asked him about the behaviour of the function

$$f(x) = \frac{1}{x}$$

in the field of complex numbers, the student's fumbled answer was that it was an analytical function (which is almost trivial) and that it had a peculiarity where $x = 0$. "And what is the name of that peculiarity?" asked Kac. It was obvious that the student would not be able to answer, and Kac took pity on him. "Look at me; what do you see?". The student took heart: "A simple Pole, sir." Of course, in English, the term for Polish is the same as the term for a polar extreme. And that is precisely the correct answer: the function $f(x) = \frac{1}{x}$ has a peculiarity, called a pole, where $x = 0$. A simple pole.

Chapter 4

Everything Else

Statistics is a science that shows that if my neighbour
has two cars and I none, we both have one.
Bernard Shaw

The mathematics we studied at school was considered basic: arithmetic, geometry and the basics of algebra and calculus. At first glance, this seemed little short of complete. Today that is not true, it is not complete. It is very difficult to get by in the world without knowing anything about statistics, coding and computing. We do not know what will be studied in schools in 50 years time, but it will certainly not be the same as it is today, in content or volume. Everything has to change so that nobody changes, wrote Lampedusa – and, boy, was he right.

What follows are intriguing fragments of mathematics, but ones that do not fit into the traditional disciplines. We are going to make way for linear programming, astro-physics, fluid mechanics, set theory and computer theory. We are going to open the door to the future.

Tycho's noses

History has treated Tycho Brahe (1546-1601) well. He was an astronomer of great moral and intellectual scope, an arrogant man with an good sense of humour, and his assistant was none other than Johannes Kepler (1571-1630). Tycho's passion was sky-high, and the precision of his measurements of celestial objects legendary. When, many years later, Kepler noticed that the circular orbits of the planets did not match Tycho's figures, he preferred to believe Tycho's result – and rightly so, because the elliptical orbits do coincide with the figures, which were victorious in the end.

When he was 19, Tycho's arrogance led him to argue with a young diplomatic nobleman, Manderupius Pasbergius or Manderup Parsberg (1546-1625), whom time has all but forgotten. The discussion intensified over a few epicycles, both parties got hot under the collar and the argument ended in a duel.

During the clash, Manderupius sliced through Tycho's nose. History tells us that Tycho Brahe's career as an astronomer was not in the least bit affected. He had two prosthetic noses made, one from bronze and putty and the other (supposedly for formal occasions) from gold and silver. They only irritated him when he sneezed, as they always fell off.

Tycho's work lasted until his death. For a long time the cause was assumed to be a renal condition, as he drank a lot of wine during a banquet and, out of politeness, he did not leave the table to urinate until the banquet had finished. But in 1996 the remains of his grave were analysed and it turned out that he died of mercury poisoning. Murder? Alchemy? Medicine? Nobody knows.

Tycho Brahe's grave in Prague.

Galilean cryptography

Few men of science have done more for modern thinking than Galileo Galilei (1564-1642) or Johannes Kepler, but their declarations were not always correct. Specifically, when in 1610 Galileo observed the planet Saturn through the lens of his splendid (for the time) telescope. he realised that something strange must have been happening around the planet. Could it be that Saturn had ears? Because what else could those 'loops' around the planet be? Satellites? Later Galileo himself identified the prominences as the famous rings.

In order to ensure his preeminence in the case of a hypothetical discovery, he told of the finding in a letter to Kepler in the form of a cryptogram (logograph is the exact term). This was something very common in those days, and was used to say something secret but without providing any clues as to its subject. Galileo told Kepler:

smaismrmilmepoetaleumibunenugttauiras

and left him on his own with his deciphering skills. The challenge, of course, is to extract an intelligible message from this jumble of letters formed from the same letters as the original, but arranged differently. After a lot of effort, and guided by certain mathematical-astronomical clues, Kepler deduced that the cryptogram meant:

Salve umbistineum geminatum Martia proles,

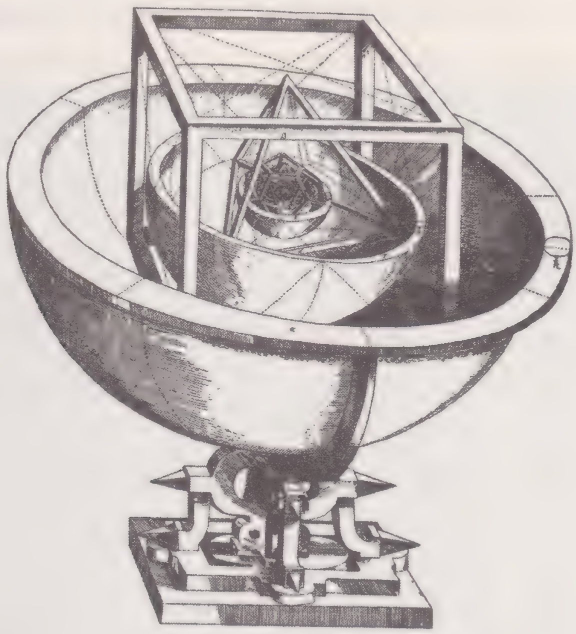
which in English would read, something like, "Hail, double knob, children of Mars". The deciphered phrase is a letter longer than it should be, but Kepler did not pay too much attention to this, as the supposed deciphered messaged coincided with his own discoveries. As you suspected (if he had been right then there would be no reason to tell this anecdote) Kepler deciphered the message incorrectly, as Galileo meant to say:

Altissimum planetam tergeminum observavi,

which translates as: "I have observed the highest of the planets three-formed." The highest of the planets was, at the time, the name for Saturn. There is a great gulf between one deciphered phrase and the other, from which we can infer that everyone deciphers such messages as they wish.

A polyhedral space

Astronomer, mathematician and astrologist, Johannes Kepler was a product of his time, a man who was sometimes a modern visionary and at other times, darkly mediaeval. One of his strangest, but most widely spread, visions was his belief that the orbits of the planets corresponded to spheres inscribed in and circumscribed around regular polyhedra. Kepler developed the subject in his *Mysterium cosmographicum,* a piece of work that features those extravagant but decorative ideas and whose illustrations focus on the regular polyhedra – also called Platonic solids.



These polyhedra, which Kepler so loved and which he studied in detail, had already been described in the ancient world by Plato, who attributed magic properties to them. Each of the triangular or square faces (tetrahedron, octahedron, icosahedron and hexahedron or cube) was linked with the substances that made up matter: earth, air, fire and water. The dodecahedron, the polyhedron with twelve pentagonal faces, was linked with the fifth element or a so-called 'quintessence' that was reserved for forming the celestial bodies. Such was the mediaeval landscape, later overcome by science. The famous quintessence become part of fairy tales, and the dodecahedron, just another polyhedron.

But then came 2003 and a somewhat mystical astrophysical article by Jean-Pierre Luminet (b. 1951) to bring our forgotten dodecahedron back. Data from the WMAP (Wilkinson Microwave Anisotropy Probe) satellite were interpreted by Luminet and his team as indicative of the fact that space had positive curvature, and its topology

was that of Poincaré's dodecahedral space. In dimension two (which is perceptible to our senses as three dimensional) this can be represented more or less as a sphere 'paved' with twelve pentagons. A few years later this geometric structure of Luminet's seems even more doubtful, but, for what it is worth, it is no less beautiful than that of Kepler.

The shopkeeper-statistician

There is consensus in considering Englishman John Graunt (1620-1674) as the founder of the science of statistics. Indeed, Graunt worked diligently in the mornings, very early, and gathered the gruesome figures of mortality (and its causes) in tables that were made public by the parish's sextons every week. Perhaps to balance things out, he also showed baptisms. He collected partial data and put it into a global and meaningful form, with a broader scope. In 1662 he appeared in the booklet *Natural and Political Observations Made upon the Bills of Mortality* which would cement his fame.

The reason for Graunt's early rising was easily explicable. He was a simple shopkeeper and he had to tend to the needs of his business, which, by the way, was a haberdashery shop. Until then it was unheard of for a shopkeeper to live a double life, so much so that Charles II himself had to publicly issue his wish that henceforth nobody impaired any trader from carrying out similar work just because their profession did not seem suitable. In fact, Graunt was named a member of the Royal Society in 1662, the same year in which his book was published. As there were relatively few people encompassed by the ruling, there was not too much paperwork, and the royal decree helped a little too.

The unfortunate astronomer

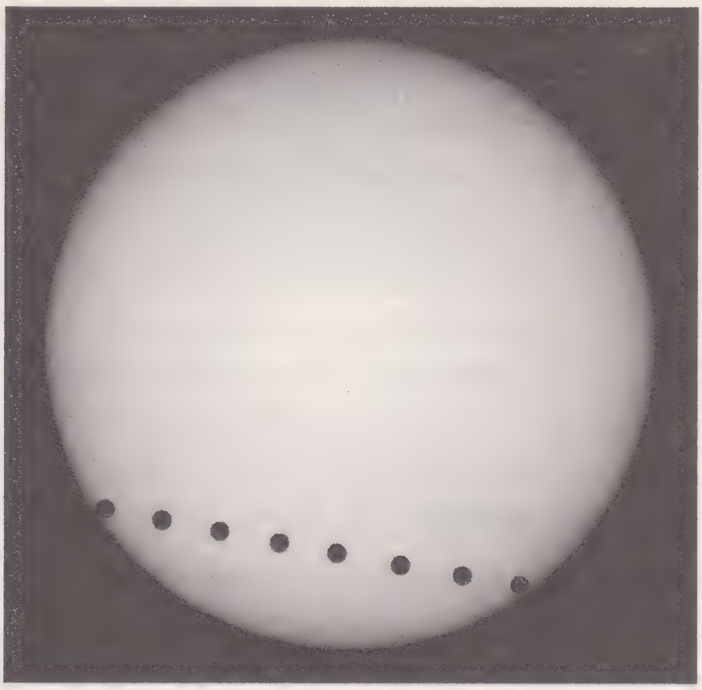
That was Guillaume Joseph Hyacinthe Jean-Baptiste Le Gentil de la Galaisière (1725-1792) whose long name barely matches the magnitude of his misfortunes. In fact, we are going to somewhat simplify his tribulations, as they are depressing and they have given rise to a drama and even an opera.

The nightmare started with a clear astronomy problem – measuring the distance from the Earth to the Sun, based on the parallax of Venus. Indeed, the passing of Venus measured at different points on the Earth at the same time gives angles of position which are also different and depend on the points chosen; the further away these points are, the larger the angles. In 1760 a group was set up to carry out the

corresponding measurements. It is worth noting that the respective positions of the Earth, Venus and the Sun only allowed periodic and very infrequent observations, with more than a century between them in some cases. Therefore, they had to make the most of any chance they had. In the coming decade there would be two moments, although separated by eight years, in which the gods would be generous, and the measurements possible.

Le Gentil, like a good French patriot, chose to take the furthest measurement from Europe in a far-lung French territory. Pondicherry, in India, was the location selected. He left for there in 1760, stopping halfway on the island of Mauritius, a French enclave in the Indian Ocean. Unfortunately, politics had already intervened: France and England were in conflict and Pondicherry was now in English hands.

After a few adventures, including a return trip to the island of Mauritius, Le Gentil decided to await the passing of Venus there. The first passing had already come and gone during his return voyage, and the conditions on a moving boat made exact measurement impossible.



Passing of Venus which took place on 8 June 2004 and in which the planet can be seen moving across the Sun (source: Johannes Schedler).

The second passing of Venus was separated from the first one, which he had already missed, by eight long years. But he was already a long way from France and it was not worth interrupting his trip. Le Gentil set sail for Manila, but the Spanish

authorities were not very welcoming. Luckily, Pondicherry returned to French hands due to a peace treaty and he headed there. He arrived, set up his observatory and waited for Venus. The weather was wonderful, even on the day before the observation, but dawn on the big day was stubbornly overcast, and it stayed that way, not allowing any observation at all. So, eight years down the drain and Le Gentil, gentle as he may have been, went a bit mad.

But, as hard as it may be to believe, things had only just got started: when Le Gentil returned to his homeland (having suffered from dysentery, among other things), he found that they had declared him officially dead. His belongings had been divided out by his heirs, he had lost his prestigious seat at the Académie des Sciences, and his wife had remarried. It took him a few years to tidy up this mess, but he eventually returned to his academic seat and married again. Fortunately he did not have to prove that he was alive, perhaps because the bureaucracy did not think of it, and also because even King Louis XVI took pity on Le Gentil and took care of his problems.

Statistics do not lie

With some justification, we do not expect writers to be good mathematicians. There are those who were, like Alexander Solzhenitsyn (1918-2008) or Jorge Luis Borges (1899-1986), but they are the exception, not the norm. However, ignoring even the most basic concepts is not completely acceptable either.

Charles Dickens' (1812-1870) refusal to take the train on a particular occasion is symptomatic. It is not that Dickens had anything in particular against the railway. The thing is that he had to travel in early December and, as sometimes happens, the accident figures for the railway were still below that indicated by the statistics for annual rail disasters. Dickens reasoned, wrongly, that this meant there would be a higher rate of accidents until the year ended. So, he did not travel. He did not consider that fortunately there would be fewer accidents that year than the previous one. His confidence in statistics must have been as unbreakable as the faith of martyrs.

If Fyodor Dostoyevsky had learnt the basics of probability properly, he never would have been a compulsive gambler, but nor would he have written *The Gambler.* And if in Hollywood they had learnt about the calculation of structures perhaps King Kong would never have been conceived. Things are better off left as they are.

The programming countess

Their are various characteristics of Ada Augusta Byron (1815-1852), later Countess of Lovelace and better known as Ada Lovelace, which concur to make her an appealing historical character.

1. She was the daughter of Lord Byron, a famous father, poet and romantic writer, although she never met him in person.
2. Her family was a perfect example of dysfunction with her parents separating during the pregnancy. Her life also took place in the Victorian era, with the values that then prevailed.
3. Against the tide of her times she chose an education in science, influenced by the fact that her mother had mathematical training and to take her mind off her dreams of following in her father's literary footsteps. Among her teachers were Augustus de Morgan, a famous mathematician.
4. She was an excellent mathematician; her friendship with Charles Babbage led to tangible results that were surprising for their time. For use with Babbage's mechanical computer (the so-called Analytical Engine, which Babbage never finished) she came up with what today could qualify as the first computer program in history, based on perforated cards. The machine was only partially built, but the program still worked.
5. Outside of science, she had a dramatic life, which included an excessive love of horse racing and gambling, including a few associations with shady bookmakers and a million-pound rescue from debt in exchange for diamonds.

She died at the age of 36 from uterine cancer, a slow and painful death. Her mother, undoubtedly sincerely worried about the spiritual well-being of her daughter, and believing eternal redemption as something that started in this world, denied her the help of morphine and other pain medication. She believed that the physical suffering, which must have been horrendous, contributed to the purification of her soul freeing her from guilt arising from the sins of adultery and compulsive gambling.

She also sought control, *post mortem*, of her daughter's papers and other objects of an intellectual nature. However, Babbage, who was an MP by now and not a man to be argued with, did not allow it, taking responsibility for them and their final use. In the 1970s a programming language was created by Honeywell Bull which was christened ADA in her honour. It is still used today and is a well-deserved posthumous homage.

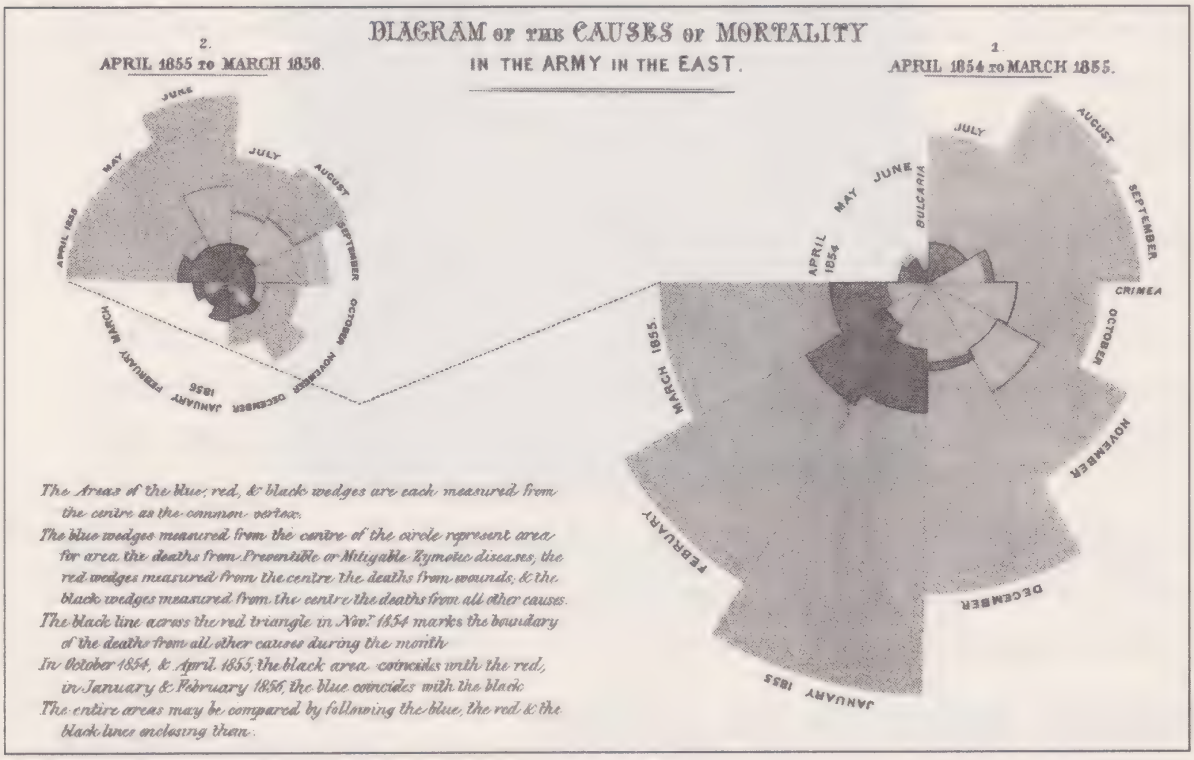
Ada Augusta Byron King (King was the surname of her husband), but better-known as Ada Lovelace, portrayed by British painter Margaret Carpenter.

Florence Nightingale and statistics

The Crimean War has inspired books, disputes, reforms, parliamentary storms and even films, notably the *The Charge of the Light Brigade* (1936) in which Errol Flynn heroically rides to glory. It was a genuine military disaster, but there was one good thing about it: the British government sent a woman to the war zone – Florence Nightingale (1820–1910) – as chief nurse. Dubbed 'the Lady with the Lamp' by the soldiers she cared for, she clearly saw all the misery, dirt and delays in the military hospitals and started her admirable crusade for public health. Once the struggle was over, she fought a much more important battle, this time in her own backyard. She won, gaining the support of Queen Victoria and the prime minister, Lord Palmerston, for her cause. Florence was unmarried and therefore it was frowned upon by society that she should choose to study mathematics (her professor was none other than James Joseph Sylvester) and, in particular, statistics. Despite everything,

she ended up being the first female member of the Royal Society, which is another unusual achievement. In her public life Florence put her knowledge of statistics to use in nursing, an activity which she revolutionised. Her statistics were replete with striking and realistic sector charts (or 'pie' charts as they are commonly called today). They were so clear that Florence's ideas were taken on, health was reformed and we all benefited.

A monument to Florence Nightingale was erected in Waterloo Place in London. The many sick whose lives she saved erected another invisible one to her.

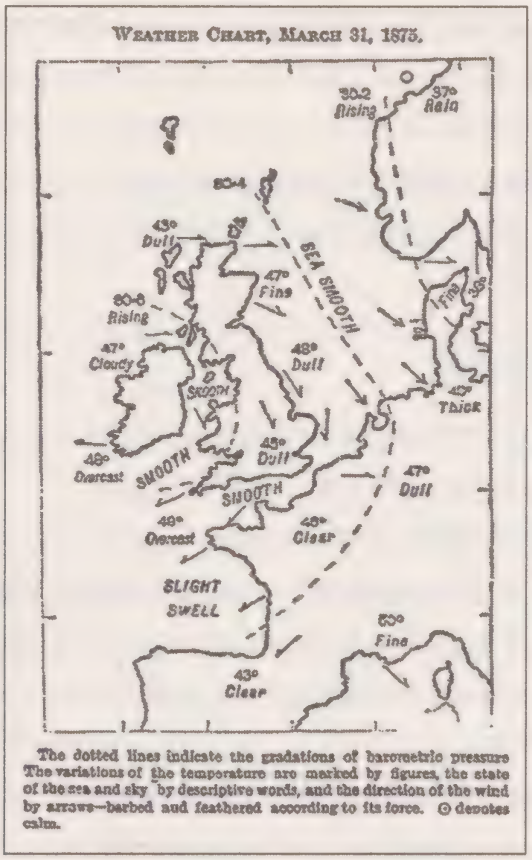


Original diagram by Florence Nightingale. It can be seen how mortality drops in parallel with the implementation of her measures.

Statistics and genocide

Francis Galton (1822-1911), Charles Darwin's cousin, was a scientifically unclassifiable character. Meterologist, astronomer, psychologist, inventor, anthropologist, explorer and, of course, mathematician, although he never studied systematically. He was one of those roving geniuses who, like his cousin Darwin, took England to the pinnacle of its scientific glory; nothing seemed foreign or uninteresting to him, from the measurement of noses to the creation of a female 'map of beauty', autosuggestion and inheritance. He was an excellent statistician and has gone down in history as

the author of the universal concept of regression and correlation. Unfortunately, he is also remembered for the controversial concept of eugenics, which he established in around 1865.



The first weather map in history by Galton, published in the Times *in 1875.*

Eugenics can be grossly defined as the eventual improvement of populations (previously races were spoken of) through the suppression of negative characteristics (we would now say negative genetic characteristics) which impede their progress. Eugenics was very fashionable for several decades, as governments and individuals (for example, the economist Keynes) looked kindly to its inherent progress; the use of eugenics (or planned genocide) by the Nazi regime completely changed that and eugenics became an abhorrent term.

On the other hand, the Hardy–Weinberg theorem, linked to Markov chains and their points of equilibrium, revealed, this time in such a way that it was mathematically sound, that in terms of heterozygous features any hope of eliminating all owners of a recessive feature was in vain. Unless all 'abnormality' (including its carriers), which were in many cases recessive and invisible, was completely eliminated, ultimately

inexorable laws of inheritance determine the regression of the characteristics in their primitive proportion.

So, eugenics stopped becoming politically correct; it was morally dubious and mathematically incorrect. However, recently, with the advent of the manipulation of the genome, the case has been opened again. It is not that the Hardy–Weinberg law does not work, but the rules of the game have been altered. It does seem possible to act on the gene that determines the recessive feature and identify the owner, even where it is not externally evident. The problem now is to act on the gene and not on the carrier. It seems that eugenics and Galton are back once more.

Tennyson and Babbage

What we have here, happily brought together by mathematics, are two completely different mentalities, two very different English talents.

Alfred Tennyson (1809-1892), better known as Lord Tennyson, was a great poet, considered the best of his time, and was a member of the Royal Society thanks to his personal interest in science, its encouragement and promotion. As for Charles Babbage (1791-1871), also a member of Royal Society, he was a philosopher, engineer, cryptographer and, above all, a mathematician who is highly appreciated by posterity as the pioneer of computational techniques. The basic concept of the programmable computer is owed to Babbage, and his failure to surpass a primitive machine, the 'Analytical Engine' as he called it, was because technical resources at the time did not allow it. There is a big difference between a steam machine and microcircuits, the same that separates the technology available at the time of Babbage from ours today.

In many ways Babbage was a strange character and was even controversial as a member of parliament. His crusade against street musicians, who he found unbearable, is a good example of this. Imagine Tennyson's surprise when he received a letter from his colleague Babbage which said:

> "Dear sir,
>
> In your beautiful poem *The Vision of Sin* the following verses can be read
>
> *Every moment dies a man*
> *Every moment one is born*

I need hardly point out to you that this calculation would tend to keep the sum total of the world's population in a state of perpetual equipoise, whereas it is a well-known fact that the said sum total is constantly on the increase. I would therefore take the liberty of suggesting that in the next edition of your excellent poem the erroneous calculation to which I refer should be corrected as follows:

Every moment dies a man
And $1\frac{1}{16}$ *is born.*

I may add that the exact figures are 1.067, but something must, of course, be conceded to the laws of metre.

Sincerely yours,
Charles Babbage"

As you would expect, the letter did not receive a reply, and the wording of the poem was not changed. How can someone be so far off with their poetic appreciation or, rather non-appreciation? Imagine someone reading

Green, how I want you green.
Green wind. Green branches.

and concluding that García Lorca was colour-blind or that he should have been more specific about the green, the radiant chromatic band with a wavelength of between 520 and 570 nanometres.

The dishonest baker

For a while, and until his death, the world mathematical crown belonged to Henri Poincaré (1854-1912). It is said that James Joseph Sylvester (1814-1897) made a trip to Paris specifically to meet him in the flesh, and he was lost for words when he met him, as if he was before a god. Such was his brilliance that in his time as a student he never took a single note. An excellent writer and philosopher and a top-class thinker, he was on the cusp of the theory of relativity itself.

His results, based on those of Lorentz and reinterpreted with the music of Fitzgerald and Minkowski in the background, brought him very close to

announcing the theory of relativity which was later developed as a body of coherent work by Einstein.

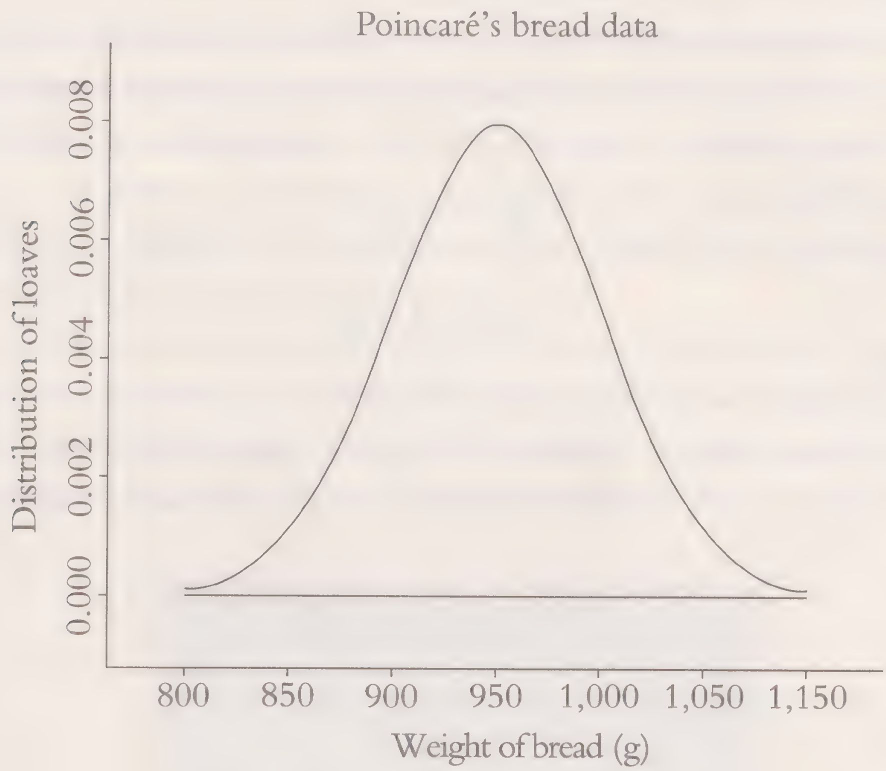
For such an esteemed figure, someone to whom we owe visionary advances in fields as diverse as chaos and topology, an incident that took place with his baker is surprising; but, as it is confirmed by the authority of the Boston Museum, it is included here.

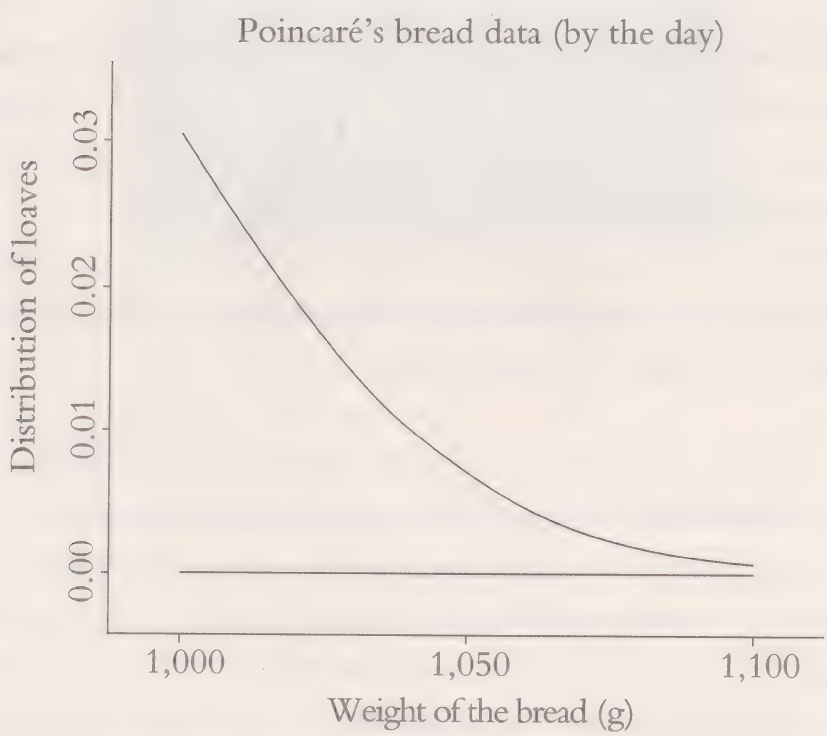
Henri Poincaré in his office.

Poincaré reported his baker, arguing that he was receiving 1 kg loaves of bread whose weights were altered and were lower than acceptable.The mathematician had taken the trouble to keep a record of the loaves received, and their weights followed a graceful curve, the Gauss bell, showing a normal distribution with an average of 950g, less than the kilogram that was ordered.

The evidence was inarguable, and the baker received a warning from the police. A while later, someone asked Poincaré about his loaves of bread and the eventual improvement in the service. Poincaré explained that their were two answers to that question: his level of satisfaction with the service was indeed currently high, as his bread now weighed 1,000g but, Poincaré continued, the baker continued to con the rest of his customers. Indeed, Poincaré's distribution curve now only showed its right-hand side, that of the loaves with a weight greater than 1 kg.The curve clearly

showed that Poincaré was only being given the loaves that exceeded the normal weight, excluding the others that were on the other side of the distribution. Never attempt to trick a statistician!





Birds of a quaternion flock together

Both Sir William Rowan Hamilton (1805-1865) and Éamon De Valera (1882-1975) were Irishmen, although they could not have shared a passport. While Hamilton always had a British one, De Valera became president of a country that was by then independent, and his passport was, at the end of his long life, an Irish one. As well as a common nationality, they shared an equal passion for mathematics. Hamilton invested many years into finding an algebraic body that generalised complex numbers; he ending up finding it, and thus invented the quaternions, in 1843.

The quaternions are formed by the combination of symbols of the type

$$a\mathbf{1} + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$$

(it is normally written $a + bi + cj + dk$, without the one), where a, b, c and d are real numbers, 1, unity and a distributive product together with the conditions $i^2 = j^2 = k^2 = ijk = -1$. The multiplication table for **1**, **i**, **j** and **k** is as follows:

×	**1**	**i**	**j**	**k**
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

The set of quaternions constitutes a body and comprises the complex numbers (considering the quaternions where $c = d = 0$ is sufficient). It is all highly ingenious and legend has it that it came to Hamilton all of a sudden when he was passing under the Brougham Bridge in Dublin.

The discovery of the quaternions seemed so marvellous to everyone and so genuinely Irish that, years later, Éamon De Valera unveiled a commemorative plaque on the arches of the bridge. The plaque states:

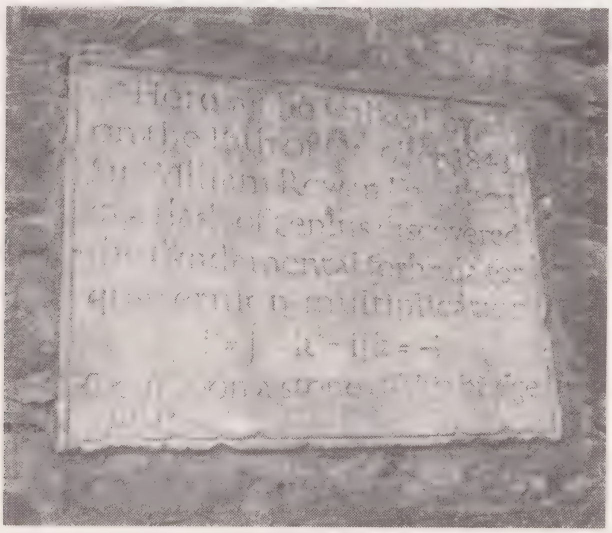
"Here as he walked by on the 16th of October 1843,
Sir William Rowan Hamilton in a flash of genius
discovered the fundamental formula for quaternion
multiplication $i^2 = j^2 = k^2 = ijk = -1$,
& cut it on a stone
of this bridge."

Everything would have ended here if Éamon De Valera (Dev to his friends) had not studied mathematics and considered himself a mathematician. In fact, in 1913 De Valera applied for a position as professor of mathematics but he did not get it. Coincidently De Valera's speciality was quaternions, and according to Arthur Conway, an ex-professor of Dev's, he "studied the subject in depth". So, in 1916 when De Valera was in jail, expecting to be executed the following morning, he proudly wrote, as an epitaph:

$$i^2 = j^2 = k^2 = ijk = -1.$$

He could not have loved the subject any more.

And so De Valera embraced politics, and mathematics lost an expert in quaternions, but it seems that he came out on top in the former. Despite a slight and dangerous pro-Hitler slip, De Valera came through World War II and became the president of an independent Ireland, the Eire we know today.



The plaque on Brougham Bridge.

The useless theorem

Set theory is an essential part of the fundamentals of mathematics, although its somewhat hasty introduction into primary curricula has led to many divisions and disputes, most of which have been overcome. Today, however, nobody argues that it has a central role in the study of science, but in the early 20th century the great discipline, mainly created by Georg Cantor (1845-1918) and Richard Dedekind (1831-1916), does not seem to have aroused a great deal of enthusiasm in some

academic fields. At the University of Princeton a committee of scholars held a meeting to discuss the study plan for mathematics. Physicist-mathematician Freeman Dyson (b. 1923) tells of a conversation between the astronomer James Hopwood Jeans (1877-1946) and topology specialist Oswald Veblen (1880-1960). As the plan was already looking a little too packed, Jeans suggested lightening its content: "We could remove set theory; at the end of the day it is a part of mathematics that will never be important to physicists," he said. Anyone who studies quantum mechanics and is up to the eyeballs in sets will bear witness to the fact that Sir James was not a great prophet.

Let's adhere to the rules of etiquette

Whatever a mathematician reads, and wherever they read it, if they see formulae they will undoubtedly understand them, here and everywhere. There is a common language in this science, and what is published in one place is understood in another, even when the latter is 10,000 km away. The symbols and abbreviations are the same, where the rest of the idiomatic expressions, those we call 'language' or 'tongue', will certainly be different (preference favours English above all, then French, Russian and Mandarin), but the heart of the matter, the 'metalanguage', will be the same. Everything which is equal is assigned the '=' sign, and when '$\in$' appears, 'belongs to' is understood. When someone with mathematical knowledge comes across expressions such as

$$\rho \frac{Du_i}{Dt} = \rho F_i - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i}\left[2\mu\left(e_{ij} - \Delta\delta_{ij} / 3\right)\right]$$

they do not flinch, they understand it perfectly, even if, at first, they do not know that it is the formula for the conservation of linear momentum in a fluid.

Many tiresome roads have been followed in mathematical thinking until reaching the current state in which a community of scientists can understand their science and express it in the same metalanguage. We should thank those who, often with great modesty, invented universal signs such as

$$+, -, \times, \sum\ , \geq, \neq, \div, \rightarrow, \in, \subset, \varnothing, \partial, \infty, \pi, (, \sqrt[n]{\ }, a^n, \int\ , \ldots$$

and agreed to use them in their work. Before these signs, these abbreviations, mathematics was a practically endless and incomprehensible discourse. Try to explain something as academic as the familiar quadratic equation

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

in words, without exponents, or letters or =, or +, or –, or the division sign, or $\sqrt{\ }$. Or the logical symbol ⇔. How are you getting on?

Many of these signs are attributed to little-known characters, such as the modest Episcopalian clergyman William Oughtred (1574–1660), whose profession was to lead mass and give sermons, to whom we owe the × to indicate multiplication and the symbols sin α and cos α (and the circular slide rule). In his life he only wrote an 88-page booklet, and in his time he was considered an amateur mathematician. This was at a time when mathematics was stumbling into adulthood.

When can we say that mathematics came of age? One answer would be: when enough mathematics books were printed in order to establish universal rules for nomenclature. Without common rules for etiquette there would be no common books. In 1875 in the UK a committee was formed for unifying the typography of printed books, their symbols and abbreviations; it was the beginning of the end. That was many moons ago, and many completely new fields and theories of mathematics have come to light, but the basic rules for etiquette were already firmly established.

Logic has its logic

Logician-mathematician Willard van Orman Quine (1908–2000) is remembered above all for his penetrating studies on the relationships between common language and scientific language. In the academic world many shared his common sense and points of view when he actively debated with Jacques Derrida and other 'deconstructors', labelling them no more than pseudo-philosophers if not charlatans. His common sense translated into a practical sense and 'Van' (as his acquaintances called him), who wrote a lot with a typewriter, changed some of the letters on the keyboard. Specifically, the '1', '!', and '?' were substituted for other special logical signs, frequent in Quine's writing.

Although it is evident that he saved time, it is difficult to understand how Quine was able to get by without something as common as a question mark. His friends asked him about this and received a typically logical answer: "You see, in my line of work, everything is about truths."

Some devilishly difficult homework

American mathematician George Bernard Dantzig (1914-2005) is known among linear programmers as the man who invented the simplex algorithm, a fundamental instrument in what has become known as operational research. In the world of anecdotes, he is famous for confusing homework with serious research articles; it is a story worth telling.

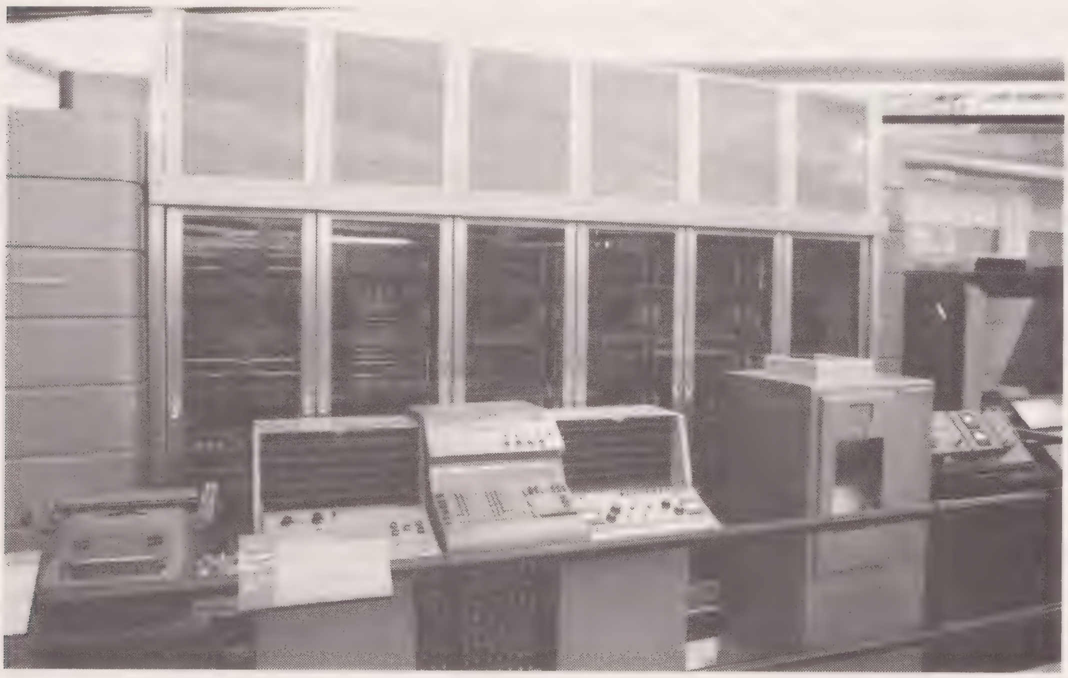
Dantzig wanted to graduate and, in 1939, he had a well-known statistics professor, the Polish-American Jerzy Neyman (1894-1981). He once arrived at class late and asked Neyman to let him copy what was written on the board, as he did not want to lose track of what had been done. On the board he found two statements which he assumed were the homework and he copied them into his book. When he got home he set about doing his homework and, wow, what an effort it required! So much so that he handed it in late to Neyman and apologised profusely. "OK, leave them on the corner of my desk," Neyman said to him, pointing at his disorganised papers. Dantzig left them there a little bewildered at the mess.

Weeks passed and one Sunday morning he heard a knock at the door. It was Neyman, who was excited and weighed down with papers. "Read this quickly, I'm thinking of sending it today for publication." It was his homework, converted into publishable articles with an introduction from Neyman. What Dantzig had mistakenly taken to be a piece of relatively difficult homework was actually the statements of two important statistical conjectures that nobody had been able to prove. He simply had not realised, and he solved them as merely complicated problems.

Everything ends in 'AC'

Friends of Hungarian-American genius John von Neumann (1903-1957) were used to calling him Johnny. Among other things, he is considered a founding father in the field of computing. The so-called 'Von Neumann' architecture sorts computers into parts – the central processing unit and the programs on one side, and the memory on the other. The trick is in integrating the program for the calculation that is going to be carried out into the central unit. Von Neumann is a contemporary of the famous ENIAC (acronym of *Electronic Numerical Integrator And Computer*), a frightful patchwork of thousands of diodes, plugs, cables and relays, which weighed nearly 30,000 kg and was capable of calculating 35 square roots to 10 decimal places in a second, something that was unimaginable at the time. And, after programming, all without human intervention.

The precursor to the computer HAL from the film *2001: Space Odyssey* was born. John von Neumann built his own version of the ENIAC, a machine he called *Mathematical Analyzer, Numerical Integrator, And Computer*, and whose acronym (Von Neumann was a real joker and told jokes in three languages!) was MANIAC. The RAND Corporation built the *JOHN von Neumann Numerical Integrator And Automatic Computer*, whose acronym was JOHNNIAC. For thirteen years, from 1953 to 1966, the JOHNNIAC worked without any hiccups, with successive improvements which made it increasingly effective. Today a simple PC is much faster and more efficient, but we are talking about 1953!



A view of the JOHNNIAC, which is currently in the Computer History Museum of California.

The theorem that was demonstrated twice

Although it may seem strange to us now, the well-known mathematician Paul Halmos (1916-2006) was once a humble assistant whose time as an independent thinker came eventually. In his youth Halmos filled the role of assistant to Von Neumann, an investigator with more years under the belt than him, as well as a genius whose intellect was difficult to match. As he tells us in his biography *I Want to Be a Mathematician*, in around 1941 he and Von Neumann set off on a joint professional adventure related to the theory of measurement and probabilities. They reached a very difficult point at which Von Neumann addressed the creation of a

pathological and very complicated set in which it was necessary to frequently recur to the continuum hypothesis through, Halmos says, "a subtle transinfinite double induction". As can be seen, testing the final theorem was delicate and intricate, even for Von Neumann. Halmos just managed to follow his reasoning... without even taking notes. Von Neumann, who noticed, warned him. Halmos, who was excited and thought he had understood everything, did not take any notice of him.

When it came to getting the theory down on paper, Halmos was horrified when he realised that he could not remember everything. How did it go again? It was impossible to remember all the steps. In a piece of truly bad luck, he was not able to speak to Von Neuman until a few days had passed.

With a timid smile he explained what had happened to the great man, and he had the rare privilege of seeing Johnny angry (he never got annoyed). Von Neumann set about demonstrating the theory again, step by step, with great difficulty. Fortunately he was able to repeat his reasoning and, a long time later, he produced the set in question and the final result, quite an achievement, even for Von Neumann. This time Halmos took detailed notes.

The icing on the cake is that Halmos stamped his name next to that of John von Neumann in the article *Operator Methods in Classical Mechanics II*. The original article, numbered with *I*, is a genuine and revered classic in physics/mathematics.

Combinations with repetition

Poet, novelist and occasional mathematician Raymond Queneau (1903-1976), who will undoubtedly go down in history for writing *Zazie dans le Métro* (as well as the lyrics to a song for Juliette Gréco) also delved into the territory of combinatorial calculus. He had already been beaten by Mozart in the world of music, but Queneau now tried applying combinatorics to poetry, which looks very difficult at first glance. A very short book, *Cent Mille Milliards de Poèmes*, with just ten pages and a sonnet on each of them, suggested a new way of constructing other sonnets (very modern ones, with a great deal of hidden meaning and adaptable to taking on any meaning at all) from a few pre-prepared verses.

All that had to be done was to take complete lines from any of the sonnets that had already been written in the existing ten-page book. There were, in total 14^{10} possibilities, even enough for the most dedicated poetry enthusiast. At the rate of a poem a minute, a calculator would tell us that we could easily compose them all in a little less than 200,000,000 years.

Another application of combinatorial calculus, this time a more imaginative one, can be found in the writing of Arthur C. Clarke, who was also a founding figure in artificial geostationary satellites and the idea of a space elevator. In *The Nine Billion Names of God* Clarke invents a computer that prints the possible names of God for some monks by means of earthly permutations. The monks believe that when all the names are written the world will end. The trouble is that when the computer finishes its work, the world really does end. Artistic licence, of course.

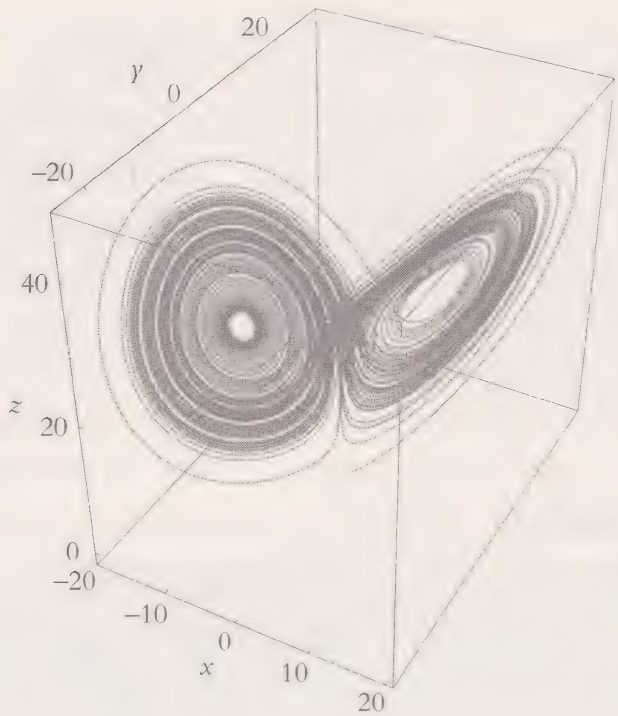
Fluttering butterflies

What is the 'butterfly effect' that appears in books? To comprehend it first we must understand chaos, a concept that is relatively new to mathematics. You could say that it all began when, in 1961, the meteorologist Edward Norton Lorenz (1917-2008) built a dynamic system that would serve as a model to help him predict the weather. Perhaps through laziness, he entered the figure 0.506 into the computer, instead of 0.506127. To his amazement, he found a general scenario that was not only different from what he expected, but it was way, way off. A tiny variation in the input data led to an enormous variation in the dynamic system. He checked it time and time again and always obtained equally stunning results; chaos theory had just been born.

A more in-depth study brought a little order to that chaos. The output data continued to be chaotic, but after following erratic and unpredictable routes, what could be confirmed is that they tended towards a set of final figures, as if the data was undergoing some kind of fatal attraction.

The set of all these infinitely long destinations is what is called the attractor. When a point moves erratically and chaotically in a dynamic system, its final destiny in infinity is another point of the attractor. The path is always chaotic, but in infinity, the limit which will never be reached, the path will die at the attractor, in the same way that many believe souls go to heaven or to hell.

The attractor has, if you will excuse the expression, an irresistible poetic attractiveness. Lorenz is the first who, when immersed in the chaos of the meteorological predictions, discovered an attractor, his famous butterfly, a set of points whose form vaguely resembles the wings of a lepidopteran and which, by the way, is a fractal set, with a Hausdorff dimension of 2.06 ± 0.01, it is a geometric wonder.



The Lorenz attractor, a three-dimensional fractal set whose shape is reminiscent of a butterfly.

The fact that this attractor resembles the wings of a butterfly is something that has aroused the imagination of countless writers and film-makers. The most famous of them is possibly the great science fiction writer Ray Bradbury, one of whose stories, *A Sound of Thunder*, describes a journey through time in which the death of a simple prehistoric butterfly is directly linked to dramatic political change in the present. Instead of a liberal president, a terrifying fascist is elected. It could not be more evocative; the simple flutter of a butterfly in a far-off scene can determine the behaviour of something in the present, something that appears to be completely unrelated. Dynamic systems can be chaotic and small causes can imply great effects. In the short-term (a small line of eternity) predetermination does not exist; chaos is an infinitely threatening shadow that leaves no room for predictions. In the long term, there is the attractor, but it is inexpressibly far-off, on the limit, the border of infinity.

The best is the enemy of the good

For a genuine democrat who votes in every election possible, and who believes that their vote contributes to modifying the behaviour of society, ideally the voting system would be perfect, obeying certain requirements that make it perfect. We know that there are many voting systems and that there should be one which is the best – a

supersystem. What should the features of the perfect system be? You can find these features, each with long explanations on the web. As there is not enough space here and the explanations are a little tedious, we are not going to copy them in full. We will limit ourselves to saying that in order to have the perfect democratic system, logically, the voting system that transfers the individual preferences of the public should include five features:

1. Non-dictatorship: no individual preference can be imposed on others.
2. Unrestricted domain: everybody should be able to arrange their preferences.
3. Unanimity: if everyone prefers the same thing, that is the final choice.
4. Positive association of social and individual values: the voting will always provide the same result if the preferences do not change.
5. Independence of irrelevant alternatives: if one option is eliminated, the rest do not change.

Winner of the Nobel Prize for Economics in 1972, Kenneth Arrow (b. 1921) carefully studied all of the above with his mathematical magnifying glass; he processed it and issued his surprising verdict. It is impossible for a system to respect all of the conditions. Some can be respected, but not all of them at the same time. "Nobody's perfect," as Billy Wilder once said.

An attention-grabbing title

American mathematician Yves Nievergelt has written about computers, wavelets and statistics. One of his articles, published in 1987, is still reprinted as a successful best-seller for students of economics and social sciences. Specifically, it is about the mathematical concept of elasticity.

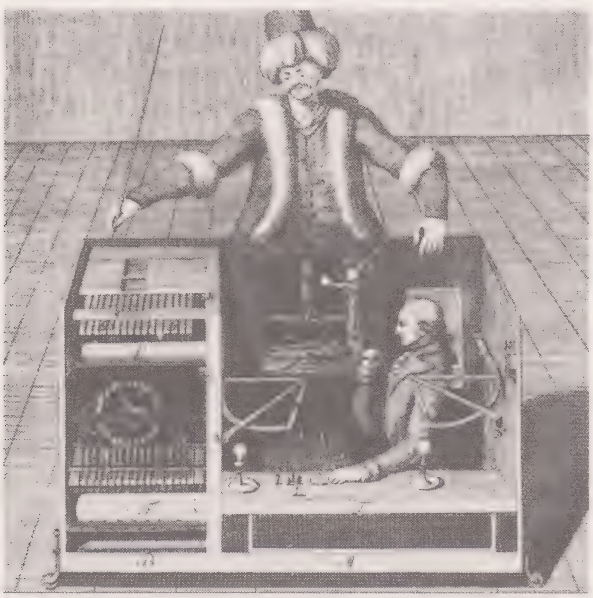
If the uninitiated should search for glamour within its pages it will be in vain, as they are full of formulae, with derivatives, logarithms and other algebraic 'horrors'. Reading through to the end, they may realise that smoking is not good for you and that anti-tobacco taxation measures have little effect on the habit, but give a great boost to the total tax take, which can then be used to solve tobacco-related problems. They will also realise that the demand for prostitution is inelastic and that pressure on the clients is more effective than pressure on the prostitutes themselves. And they will learn that the demand for salmon depends, among other things, on their relative abundance, the survival of the eggs and of young salmon, etc. You can

learn an awful lot about a huge diversity of things, all of which are difficult to study because obtaining data on these subjects is not common.

And the title of Nievergelt's informed mathematics article? *Price Elasticity of Demand: Gambling, Heroin, Marijuana, Whiskey, Prostitution, and Fish.* Attention-grabbing? You could say that.

A game that is more than a game

If there is a king of games, it is undoubtedly rudimentary chess. There is no element of luck and pure strategy is the preference. And memory, because the number of possible moves in a given match is around 10^{123}, an astronomically large figure. A few professional mathematicians, such as Emanuel Lasker (1868-1941), figure on the list of world champions. And we are just referring to the conventional flat chess with 64 squares, because as far back as the Victorian era, mathematician Arthur Cayley (1821-1895) was already considering three-dimensional chess. It is not such a crazy idea – after all the game is played in *Star Trek*.

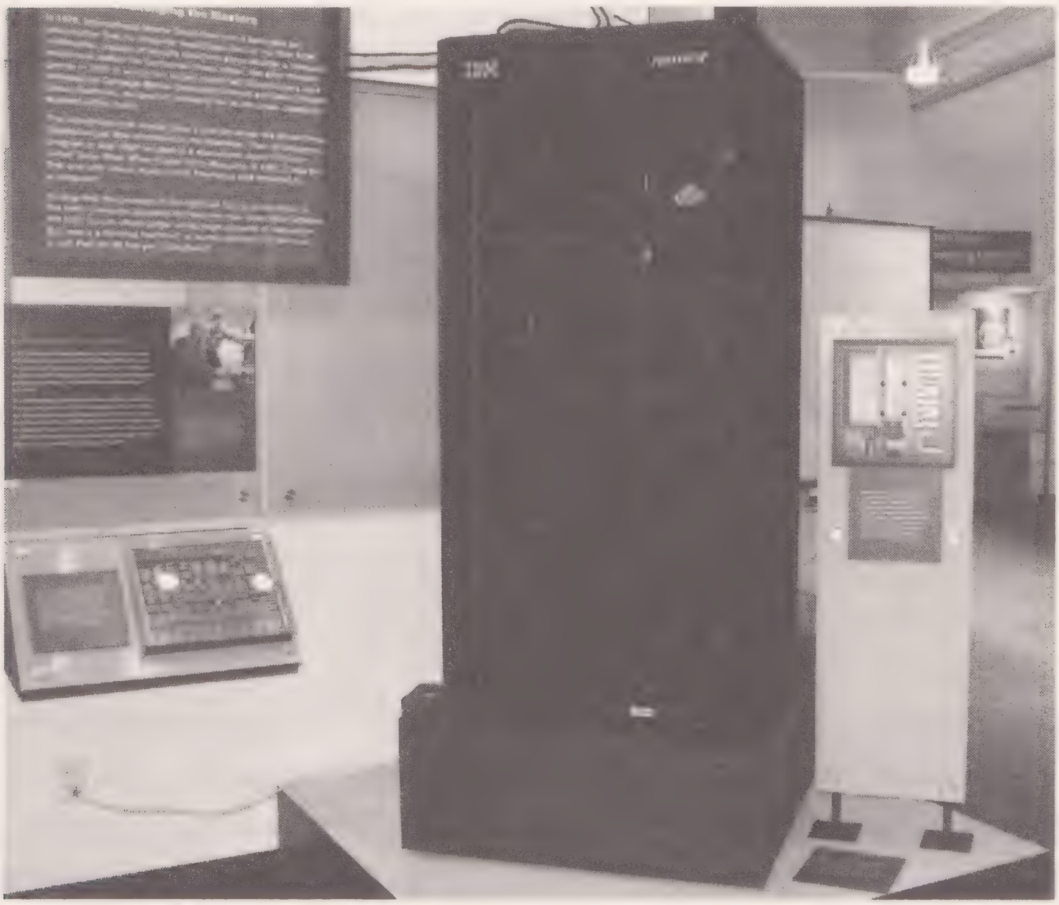


The automaton called 'The Turk', which caused an uproar in the world when Hungarian engineer Wolfgang von Kempelen introduced it in 1769. It seemed to be able to play chess, but it was actually a well-conceived con; a normal-sized person could hide inside the mechanism.

Nobody has yet been able to make a suitable study of chess, as for now its complexity makes it too vast, even with the modern sophisticated weapons of game theory. This has not prevented a few valiant attempts, such as that of Spanish engineer

Leonardo Torres Quevedo (1852-1936), who invented an automaton in 1914 which he called *el ajedrecista* (the chess player), which could always reach checkmate with three pieces (a king, one of his rooks and the other king). It seems a long way from the goal, which is to always win at chess played with a full set, but you have to start somewhere.

Chess is good territory (a starting place, if you will) to settle the eternal battle of man vs. machine. We know that chess playing programs are becoming increasingly better, and the morbid fascination of putting advanced software up against a gifted human mind is irresistible. In 1996 a one-off match was held between a machine, the Deep Blue computer built by IBM and world champion Garry Kasparov. Kasparov won three games to nil. So that year was still a year of human superiority.



Deep Blue, the first chess-playing computer to beat a world champion.

The following year the bitter battle was repeated; Kasparov was practically the same being, but Deep Blue (or rather, its software and memory) had improved. This time the machine won in the deciding game. Kasparov was not convinced and alleged

that there had been human intervention with the Deep Blue program during the match. IBM, as you would expect, denied it and having achieved its objective that one of its machines had beaten a man, went on to dismantle the computer. The competitions (with other machines in Deep Blue's place) continued in 2000 and 2003 and ended in draws. It is very possible that this tradition will continue.

Looking to the future, the fact is that developments until now are unimportant: the world will only remember that technological advances have already led to machine beating man once. It had to happen sooner or later. But the truly important thing is answering the question "Is human thinking similar to that of a machine?" That we do not know, and maybe it will take us a long time to find out. And maybe we will never know and it will be a Gödelian problem. And perhaps, in this case, we know we will never know.

Chapter 5

Mathematicians From the Distant Past

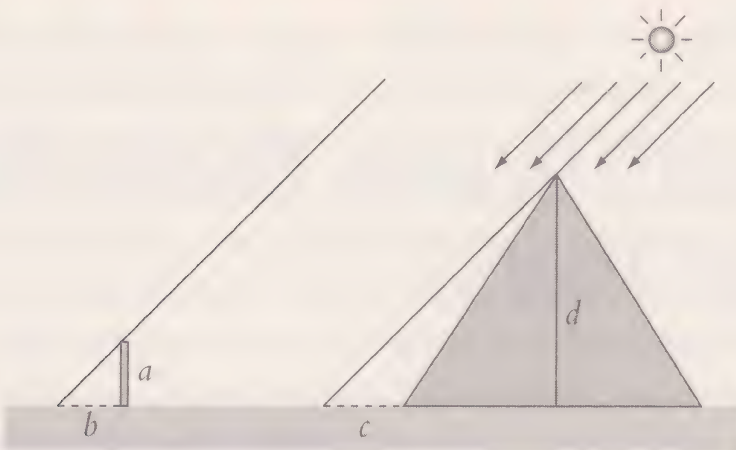
A mathematician is a blind man in a dark room looking for a black cat that isn't there.
Charles Darwin

These are minds from distant times, and time can distort our perception. Mathematicians from the past deserve a great deal of respect and admiration from us. They did not reason like we do, with efficient and universal notation, they had no magazines or Internet, the circulation of their ideas could sometimes take decades (if they were lucky) and they were rarely considered anything more than odd curiosities.

The passing of the centuries shaped the binomial, mathematical-being into something super-intelligent and psychoanalytical. The mathematician emerged from that period as a somewhat alien creature, strange by definition, highly gifted and very, very clever.

The first speculator

One of the Seven Sages of Greece, according to Pausanias, was Thales of Miletus (ca. 639 BC–ca. 547 BC), a polymath whom we will dub a mathematician for our purposes. According to Euclid, he was the first to prove a few geometric truths that may seem basic to us today, but in their era they were not, such as, for example, that there is a straight line called a diameter that bisects the circle, that the equal sides of an isosceles triangle determine equal angles, and that the angles called alternate internals are equal. And also the famous theorem that bears his name and which, it seems, allowed him to measure the height of a pyramid by measuring its shadow simultaneously with that of a stick, as shown in the following illustration:



Applying the theory that bears his name, Thales calculated the height d *from the height of the stick* a *and the length of the shadows,* b *and* c*:*

$$d = \frac{a \cdot c}{b}.$$

He was no ordinary man, and the Greek chroniclers attribute various achievements to him, all of which demonstrate his supposed ingenuity. The only one that we are going to include here, as it concerns applied mathematics, is the one regarding olives. According to written accounts, Thales had astronomical knowledge (or rather, astrological) and he occasionally used it to his personal advantage.

He predicted, by contemplating the course of the stars, that in a particular year there would be a great harvest and proceeded to hire the region's olive oil mills. When the great harvest arrived at the mills to be converted into oil, Thales had already monopolised them and he got rich in the blink of an eye. A story for entrepreneurs that shows that you can never know too much, not even in ancient Greece.

Combating fraud

The great Archimedes of Syracuse (ca. 287 BC–ca. 212 BC) is not only well-known for running around the streets naked shouting "Eureka!", but also for his geometric discoveries and his inventions that reveal his unequalled talent in engineering. He was also one of the first fighters in the struggle against scientific fraud that we are aware of. As Archimedes himself explains in his treatise *On Spirals*, the precocious genius had the habit of writing to his friends in Alexandria with descriptions of his theorems and discoveries, without the corresponding demonstration, of course, so that they could keep themselves amused by sharpening their wit and proving the

theorems themselves. It was like a gift in which Archimedes only offered the first part in order to stimulate the neurones of his colleagues.

However, some of them, committing fraud and abusing his trust, published some of Archimedes' theories as their own, in many cases without even bothering to look for proof. "If Archimedes says so," they must have thought with outrageous cheek, "then it must be true." Archimedes was furious and from then on he adopted a diabolical insurance against fraud. He continued to send the theorems, but always slipped some false statements into them. That ensured that anyone attempting to show they had proved it first would be as Archimedes said, "...presumed to know everything, but without proof, could be accused of discovering the impossible."

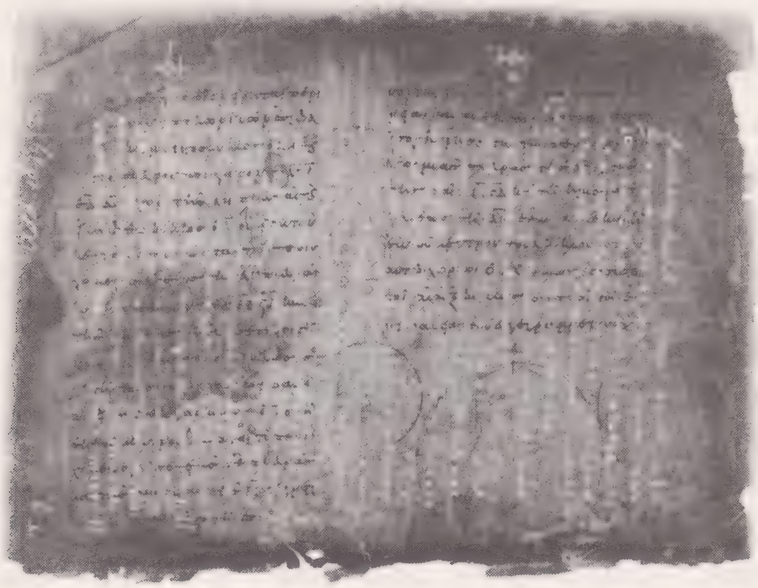
Archimedes was so fond of his discovery of the relationship between the areas and volumes of a cylinder and an inscribed sphere that he even asked it to be reproduced on his grave.

Put a palimpsest on his life

Centuries ago, parchments made with sheep and cow skin or the like were a rare asset, and anyone who found one would not hesitate to reuse it by deleting what had already been written and writing on top of it. The result was a palimpsest. In 1906, J.L. Heiberg, a Danish historian, professor of the University of Copenhagen, discovered one of these palimpsests in a Turkish monastery. It dated from the 8th century and he managed to remove the Orthodox prayers and other liturgical matters written on top of a previous text (175 pages of parchment!), an apparently heavy-going

text from the 10th century, some time around 975. The text beneath certainly did not save the souls of its devoted readers, but it did constitute a great mathematical discovery. It belonged to a lost work by Archimedes called *The Method of Mechanical Theorems*. It was completed by other documents from the wise Greek, all of which are very interesting. In the work, perhaps his last, Archimedes used concepts which are very close to our modern integral calculus (Eudoxus' method of exhaustion, together with mechanical considerations and others on the centre of gravity of bodies) and demonstrated his unequalled genius.

However, the adventures of the palimpsest did not end there. It got lost again, only to reappear 90 years later for auction at Christie's, where a multimillionaire purchased it for around $2 million dollars and then deposited the palimpsest at The Walters Art Museum in Baltimore, where fortunately it has been restored (to the extent allowed by current techniques) and it seems to be safe.



A page of the Archimedes palimpsest (source: The Walters Art Museum).

The Laird of Merchiston

This title belongs to mathematician, astrologer and necromancer John Napier (1550–1617), popularly known as the 'Marvellous Merchiston', and one of the most revered names in history – we are talking about the man who introduced logarithms!

Like many characters from the 17th century, Napier cannot be judged with our usual criteria, as neither he nor his contemporaries shared our current vantage point,

which is softened by a somewhat selfish state of affluence. The famous Samuel Pepys, for example, considered his Parmesan cheese to be his most treasured belonging (he wrapped it up and buried it during the Great Fire of London).

Napier won his place in mathematical history with his discovery of logarithms, named Naperian in his honour, which are expressed in base *e*. Many of his scholarly contemporaries, particularly the Catholic ones, appreciated his intricate and ingenious calculations of the date of the end of the world. Napier placed it, based on data from St John's Revelation, in the period between 1688-1700.

One of the first Naperian abacuses consisted of a little box which contains the so-called 'Napier bones'. The bones gave their name to the abacus that would have made calculation easy because it used logarithms.

Less grotesque, but perhaps just as incorrect is an anecdote which is often told regarding the laird and his black cockerel. Napier had good reason to believe that one of his servants was stealing from him. It is said that he lined them up in the dark and, brandishing his black cockerel like a diabolic messenger, he ordered the servants to stroke its back, stating that the cockerel had made a deal with Satan and that it would alert him to the guilty person as soon as it felt the contact of their hands. What nobody knew was that Napier had previously spread the cockerel's black back with soot from a lamp. When Napier examined their hands, they were all black from the soot; all except one of them, the presumed guilty party, who had pretended to stroke the cockerel in the dark, but had actually not touched it. If it's not true, it's well conceived.

Mathematicians are crazy

Astronomers, mathematicians and people of science in general have aroused a sense of rejection among people of the arts. If we also consider that the literati is known for its misanthropy and a keenness for writing to everything and everyone's detriment, we can understand why it does not take well to people of the sciences. Jonathan Swift (1667-1745) comes to mind in this respect. He was an Irish writer characterised by a penchant for meddling in everyone's affairs (in his literature and elsewhere).

In a famous book, Swift suggested eating Irish children. The measure may have seemed ferocious to well-meaning English society, but Swift argued, with a certain inhumane derision, that it would resolve both the overpopulation of Ireland and solve the island's food crisis. From a numerical point of view he was very right. Monty Python, with less delicacy, also upheld similar ideas in one of their sketches from *Monty Python's The Meaning of Life.*

It is not surprising that astronomers, mathematicians and technologists in general were the victims of one of his novelistic fantasies in *Gulliver's Travels.* In it is the island of Laputa, inhabited by a middle class with a passion for mathematics and everything related to it, but which completely fails to make any practical use of its knowledge. Their clothing is misshaped as it is only made from geometric shapes; their buildings are badly built; their food is not very appetising, but its shape is reminiscent of Euclidean figures; their astronomical discoveries are very accurate but useless; their inventions are absurd, etc. Even "If, for example, they were to praise the beauty of a woman, or any other animal *(sic)*, they describe it by rhombs, circles, parallelograms, ellipses, and other geometrical terms"; the result is that it does not exactly make you long to live there. And, let's not forget the name of the island; *la puta* means 'whore' in Spanish. Perhaps the most entertaining is the way in which the Laputans approach theorems. The teachers make the students ingest and digest them, literally. All this results in giving the reader the impression that they are better off avoiding mathematicians. Some experts of the work of Swift see Laputa as a somewhat brutally distorting mirror of the Royal Society, the official 'knowledgeable' society of Swift's time.

Scientific precociousness

Mathematics has an abundance of cases of precociousness, whatever period you look at. While in the past Pascal, Gauss and Fourier amazed their peers, in the

present the names Norbert Wiener (1894–1964), Enrico Fermi (1901–1954), Paul Erdös (1913–1996), Edward Witten (b. 1951) and Peter Shor (b. 1959) remind us that precocity is a recurring phenomenon. Of course there are examples of the fact that precociousness is not a necessary condition to be a mathematical genius of the future; the clearest examples are Newton and Einstein, who showed no special talents during their childhood, or even their youth. We know what happened later.

A paradigmatic but little known case is that of Pierre Bouguer (1698–1758), who the experts consider to be the father of hydrography. He was also the inventor of the heliometer, an instrument for measuring the diameter of stars. Bouguer is not well known by the man on the street, but there is a lunar crater that bears his name, and, as if that was not enough, another one on Mars; scientists do seem to appreciate him.

Bouguer succeeded his father in the position of professor of hydrography, a fact that on its own is not particularly interesting. What is interesting is the year in which this happened, 1713; Bouguer was only 15 years old.

French mathematician and astronomer Pierre Bouguer.

For the honour of the human spirit

There are many Romantic-Platonic statements about the uselessness of mathematics. So why do we practice it? Carl Gustav Jakob Jacobi (1804–1851) coined the expression "For the honour of the human spirit", an elegant way of saying that mathematics is studied for the pleasure of reasoning. An ardent defence of the pleasure of practising uselessness was put down on paper by G.H. Hardy (1877–1947), in *A Mathematician's Apology*, a famous and beautifully stylish essay. Hardy maintained that he was very proud of being a specialist in the theory of numbers, a branch of knowledge which he believed (naively) would never, ever, be of any use. But the most beautiful anecdote on the subject counts Euclid (ca. 325 BC–ca. 265 BC) as its protagonist, according to Stobaeus. A relatively new student of Euclid's asked him after a class: "And what will I gain by knowing all of this?" Euclid turned to one of his slaves and said: "Give him some coins, it is clear that he has to make some money out of everything he learns."

A popular mathematician

The great Leonhard Euler (1707–1783) was of Swiss origin and divided his professional life between Russia and the Prussia of Frederick the Great. Both nations met in armed conflict during the so-called Seven Years' War, and during the course of the campaign, the Russians demolished an estate of Euler's. News of this reached the ears of the general of the Russian troops, who it appears knew of Euler's fame as a scholar. He said he sought to fight with the enemy but never against science and generously compensated Euler for the destruction of his property. While Euler was still licking the wounds of his levelled estate, the empress of Russia also received news of what had happened and personally compensated him immediately. There was a time, it seems, in which it felt good to be a mathematician and in which collateral damage was paid for. Euler may have hoped for the fighting to continue.

Euler and Diderot

Leonhard Euler had 13 children, he was blind in one eye for 47 years, and in both for the last 21 and he wrote about 800 pages a year. He also had a phenomenal memory and capacity for calculations. One night when he could not sleep he calculated the sixth powers from 1 to 100 and he could remember them perfectly several days later!

He was also a workaholic, and his works were stacked up in the St. Petersburg Academy and then published in reverse order as each one was removed from the pile. Euler was so prodigious (and the Academy so slow) that he wrote faster than his worked was published. The result was that Euler apparently published the opposite way round to everyone else, with the most advanced articles appearing before those in which he introduced the discovery, in a kind of journey through time. To the outside observer his progressive blindness seems like it must have been a source of distress, but although Euler cannot have liked progressively going blind at all, he was never depressed about it either. When he lost the sight in one eye in his youth, he said: "This way I will have fewer distractions," and continued working.

Portrait of Euler, painted by the Swiss Emanuel Handmann in 1753 in which he is shown to be blind in one eye.

Euler is a source of anecdotes, stories and wordplays about mathematics, and this is not the only one, but it is one of the best known. Fortunately, they are so popular that there is no need to tell them all here. It is interesting to know that serious doctorate studies have been made, not on Euler as a person, but on his anecdotes. Here is one of them which is already famous.

Dieudonné Thiébault tells us in his memoirs that Denis Diderot (1713–1784) visited the court of Empress Catherine II, where Euler resided. Euler was a protestant and therefore a believer, while Diderot was at the time (he suffered from religious fluctuations) an atheist. As you might expect, a philosophical debate was prepared

in the court over their points of view. Euler began by saying: "Sir,

$$\frac{a+b^n}{n} = x,$$

therefore God exists. Answer please."

Diderot did not know how to defend against this mathematical volley; according to Thiébault he was completely ignorant of mathematics, such that he remained silent, much to the amusement of the rest of the court. He shortly decided to return to Paris.

The above is the anecdote. The reality, borne of subsequent studies by the American Mathematical Society, throws a few shadows on the facts, as much as Thiébault's memoirs may enjoy well-earned fame for veracity and equanimity. First of all, his story was altered during a transcription and it appears that events did not unfold exactly as told in the original source. Also, it is not true that Diderot was ignorant in mathematics. Although he was not a professional, he had already written some significant memoirs on several subjects. On the other hand, anyone would have been thrown off by Euler's argument. It is a perfect nonsense, and even more so coming from the world's best mathematician. And finally, it is not surprising that Diderot would make the decision to leave. Apart from other reasons only known to him, the best thing to do in a glacial place full of Russian aristocrats who laugh at you is return to a warm and cosy Paris.

Don Giovanni the mathematician

Scientists are not painted as particularly seductive beings. Think of the immortal Professor Calculus in the Tintin comics, scatter-brained, ageing, short and definitively unathletic. Their minds are a different matter, they are described as very intelligent and always with a prodigious memory. Paul Erdös, who fitted this description to a tee, remembered the telephone numbers of every single one of his many friends by heart.

Although it is true he only addressed one person by their Christian name. That was Tom Trotter, whom Erdös called Bill. The slightly stereotypical image of a scientist is a somewhat puny person, hopeless at anything sporty and, generally, lacking any sex appeal. Naturally, this is not realistic, and there have been tall and short, athletic and ungainly, attractive and ugly, seductive and repellent mathematicians. There have been all sorts.

What everybody forgets is that Don Juan, the king of seducers, was a mathematician. Among other things too, one man – a handsome Venetian – was a sociologist, spy, businessman, gastronomist, cabalist, violinist, procurer, theologian, lawyer, gambler, military man, con artist, dancer, diplomat, politician and of course, a writer. These are the strings to the bow of one Giacomo Casanova (1725-1798), whom few associate with a science as supposedly arid and intellectual as geometry.

Casanova wrote many generally mathematical texts, such as the fantastic epic *Ikosameron*, and serious papers on the duplication of the cube. The readers' interest, however, has always fallen on his memoirs, and in particularly his exploits as a lover. He did not have as many affairs as the *mille e tre* in the opera about him by his friend Mozart (it seems that there were only 126, all studiously recorded), but it is a significant figure nevertheless. So there was at least one seductive mathematician.

Giacomo Casanova portrayed by his brother Francesco.

The mathematical minister

Pierre Simon Laplace (1749–1827) reached the heights of the French administration under the reign of Napoleon Bonaparte, who made him interior minister. He should never have done so. Laplace demonstrated shortly after starting work as a member of the government that he was as good a mathematician as he was a terrible public figure – and his incompetence for politics had its consequences. Six months after appointing him, Napoleon had to dismiss him. Now in exile on the island of Saint Helena, Napoleon took the necessary time and rest to qualify Laplace's contributions to the government, not without irony:

> "Geometrician of the first rank, Laplace was not long in showing himself a worse than average administrator; from his first actions in office I recognised my mistake. Laplace did not consider any question from the right angle. He sought subtleties everywhere, only conceived problems, and finally carried the spirit of 'infinitesimals' into the administration."

A rather harsh criticism from someone who knew men as he did. It is interesting that Napoleon should reproach someone for looking so hard at the trees that he could not see the wood, when in his books this was not the case. Biot tells us that Laplace frequently used the expression *"Il est aisé de voir que…"* (meaning "It is easy to see that...") when he knew very well where he was headed but was too lazy to go into details for the benefit of his readers.

Laplace died a nobleman. When Napoleon fell he defected to the Bourbon side and was named Marquis de Laplace.

In search of the lost formula

Physicist and mathematician André-Marie Ampère (1775-1836) was one of the founding figures of electromagnetism. The ampere, or unit of electrical current, was so called in his memory. The man himself was a somewhat forgetful character; as we are about to see he fitted the stereotype of the scatter-brained intellectual. It is said that he once unleashed a heated argument on a visitor to the Collège de France without realising that he was fervently arguing with a man, who was unknown to him, who responded to the name of Napoleon Bonaparte.

Once, Ampère was struck by inspiration while travelling in a hired horse-drawn carriage and he immediately wrote the fleeting thought down so that he would

not forget it. However, he forgot where he had written it and could not find his note anywhere. In the end, and by deduction, he had to accept the evidence: he had not made his notes on a piece of paper, but on the hired carriage itself, which was going around the city transporting passengers who were completely unaware of the fact that it was also carrying with it, on its bodywork, the most intricate secrets of science. Ampère had no choice but to examine the horse-drawn carriages, one by one, until, luckily, he found the inscription he was looking for – 'informed' graffiti if there were such a thing.

The prince of mathematics

This is what contemporaries of Carl Friedrich Gauss (1777–1855) called him (albeit in Latin, *Princeps mathematicorum*). He was undoubtedly one of the greats of science, and he has had dedicated to him, for example, an asteroid, a lunar crater and a bank note, as well as several postage stamps. He was a legendary man from a humble background, incredibly precocious and intelligent and truly a child of his time. For example, he gave his sons the cane and was opposed to them practising sciences for fear that the reputation of excellence of the Gauss name may not remain at the highest level. His reaction when he was told that his wife was dying will give you an idea of his curious concept of family: "Ask her to wait a moment – I am almost done," he said.

English mathematician John Wilson (1741-1793) conjectured from his reading of Arabic authors that

$$(n-1)! \equiv -1 \pmod{n},$$

but, despite his efforts, he was unable to prove it. In the end he stated that new notation would be needed in the theory of numbers in order to tackle it. The first proof was obtained by Lagrange, but Gauss proved that

$$\prod_{\substack{k=1\\(k,m)=1}}^{m} k \equiv \begin{cases} 0 \pmod{m} \text{ if } m=1 \\ -1 \pmod{m} \text{ if } m=4, p^{\alpha}, 2p^{\alpha} \\ 1 \pmod{m} \text{ in other cases} \end{cases}$$

which is a much more general result than that sought by Wilson. Best of all, Gauss found his result a few minutes after discovering Wilson's conjecture. His comments on Wilson's efforts were, at best, scathing: "What Wilson needs is not new *notation*, but a certain *notion* of what he is talking about."

Gauss' demonstrations were always impeccable, sometimes surprising from an intellectual point of view, as he himself made sure to hide the sources of his thinking, the path taken to arrive at the final theorem. Niels Abel said that Gauss behaved like a fox, erasing his own footprints with his tail.

The anti-employee

The introduction of hyperbolic geometry, one of the non-Euclidean geometries, is owed to Russian geometrician Nikolai Lobachevsky (1792-1856), the scientist who in 1972 gained celestial immortality when an asteroid was named after him.

Lobachevsky was a model employee and a tireless worker. He was assigned by the Russian government to give classes at Kazan University, and unwittingly managed to accumulate jobs, one after the other, whether due to the illness of others or through administrative decisions. At one point in time, as well as giving classes, he was responsible for running the museum, the library and even the university observatory, and he also fulfilled the role of general director of studies. Given all of this it is not surprising that, in the end, they named him rector of the establishment.

It is said that he did not hesitate to roll up his sleeves and help out with everything. There was nothing that he thought was below him, and he was happy to take a cloth and start cleaning museum pieces. Once, an illustrious visitor entered the museum and asked a busy caretaker to show him round. To his surprise, the caretaker knew everything, showed him everything, and left him aghast at his friendliness and knowledge. Such was the impression made on our visitor (some say it was a diplomat) that he wanted to thank the caretaker with a tip. To his surprise, the caretaker was offended and rejected it. You will already have guessed that the so-called caretaker was Lobachevsky, the rector. Shortly after, the visitor attended an official reception and found out, to his embarrassment, that his supposed caretaker was the university's head authority, who, as you would expect, was among the guests... this time smartly dressed.

In order to thank him for such dedication and as a demonstration of his appreciation of one of the world's best geometricians, the Russian government fired Lobachevsky from the university as soon as he began to suffer from problems with his sight. He died blind, poor and with no official job.

While the Tsarist leaders treated him badly during his life, years later American music deal him another blow. In the far-off US, singer of comedy songs Tom Lehrer

wrote one of his hits – the song *Lobachevsky* – in which the famous Russian mathematician, joke after joke, does not come out too well. This is what you get for being a good employee.

Nikolai Lobachevsky.

A mathematician at West Point

Ferdinand Hassler (1770–1843) was a geometrician and topographer of Swiss origin, and was as frank and relaxed as the following story suggests. Hassler was invited to emigrate to the United States and fill a place at the illustrious West Point military academy. One fine day he was called by the secretary of the treasury, who it appears wanted to make savings: "Sir," he said, "your salary is enormous, almost as high as that of the Secretary of State, and it should be reduced." "True," came the answer, "but the President of the United States could make even someone like you Secretary of State if he wants, but nobody can replace a Hassler." And that is how the interview, and the matter, ended. Hassler continued to receive his high salary in full and the secretary of the treasury, of course, was not assigned to carry out Hassler's work.

Mathematical, but naive

Michel Chasles (1793–1880) was an excellent French geometrician and analyst, but they poked fun at him like any fool.

In 1867 he proudly submitted some letters from Pascal to the Academy of Sciences which demonstrated, without doubt, that the Newtonian ideas on universal attraction had been anticipated by the French genius, long before the son of perfidious Albion announced them. Quite a scientific event. Continental Europe had finally landed a mortal blow on the Anglo-Saxon island!

The letters were sold to Chasles by a Mr Vrain-Denis Lucas, who continued to provide him with increasingly interesting documents. For example, he sold him (in boxes, as so much material required space) letters from Julius Caesar to Vercingetorix, from Julius Caesar to Cleopatra, from Alexander the Great to Aristotle, etc. He even sold him a letter from Mary Magdalene to Lazarus. Chasles spent around 140,000 Francs in total.

The indisputable fact that much of the material was written in French and on paper should have aroused the suspicions of the buyer, but some people have selective hearing, and Chasles – great mathematician though he was – did not seem to apply the same criteria of rigour to the world of ideas as he did to the prosaic world of everyday reality.

A deceptive title

James Joseph Sylvester (1814–1897) was an English mathematician and fan of puzzles, who taught for many years at Johns Hopkins University in the United States. He contributed much to mathematical progress in the New World, but in the mathematical universe most people know him as the inseparable colleague of Arthur Cayley (1821–1895). One of Sylvester's characteristics was his well-known oversights regarding whether or not something had already been demonstrated. W.P. Durfee said that he even went as far as calling one particular proposition absurd... it turned out it was his own and he himself had elevated it to the category of a theorem.

A man with unique and fiery temperament, in 1858 he published an article in the *Philosophical Magazine* which must have caused a real scandal in Victorian society, at the same time as Darwin and his *On the Origin of Species*.

Sylvester's article was called *On the Problem of the Virgins and the General Theory of Compound Partition*. Its content, as anyone outside the world of mathematics would

have found out if they read it, did not include anything sinful or provocative, apart from the title. The supposed virgins are not what you might imagine, because to an algebraist the 'rule of the virgins' is an algorithm in advanced calculus, also called the 'rule of Ceres', among other mythological denominations. This dates back to times before Euler, and the exact origin of its striking name is unknown. It relates to the problem of resolving a system of two equations in the field of natural numbers; it belongs to the difficult terrain of algebraic combinatorics, and specifically the calculation of partitions.

The content of the articles with alleged sexual nature was, in 1858, laughable. If the English had had access to articles from 150 years later, they would have been able to read in the *New England Journal of Medicine* the very serious article *The Spermicidal Potency of Coca-Cola and Pepsi-Cola,* and Sylvester and his sense of humour would have seemed somewhat basic.

The swordfighting rector

German mathematician Karl Weierstrass (1815–1897) is universally recognised as one of the fathers of logical rigour in infinitesimal calculus. Modern mathematics students curse his name under their breath because Weierstrass was responsible for the famous definition of the limit of continuity, with his δ and ε:

> "It states that:
> $f(x)$ tends to y_0 when x tends to x_0, if for every $\varepsilon > 0$ there is $\delta > 0$ such that $|f(x) - y_0| < \varepsilon$ for each x where $|x - x_0| < \delta$."

Weierstrass' youth was not particularly edifying. Tall and athletic, a great beer drinker and very passionate about mathematics, he became a first-class sword fighter. In his time it was normal for students to meet at fencing clubs, challenging each other and proving their masculinity for everyone to see. The protection they used stopped them from being seriously injured, but their cheeks were not covered and so they were a logical target for blows. Having cheeks with scars was considered elegant and virile. Weierstrass was such a good fencer that he was never injured; years later photographs show us his impeccable cheeks.

The life he lived, replete with beer and comforts, prevented him from getting a university degree and doctorate as he hoped. All he had was his fencing and a secondary diploma for working as a school teacher. He taught

physics, mathematics, history, languages, geography, P.E. and, believe it or not, calligraphy.

Years later, when his mathematical merits elevated him, he became rector of Berlin University and knight of the order 'Pour le Mérite', the highest honour in Germany, established by King Frederick II. Even his doctorate was abnormal, as he was awarded it as an honorary title by the University of Königsberg.

Karl Weierstrass painted by Conrad Fehr.

Don't let her take her hat off, she's very dangerous without it

This statement, midway between sexism and admiration, is an assessment by the famous chemist Robert Bunsen (1811–1899) of the beautiful Russian mathematician

Sofia Kovalevskaya (1850–1891), one of few women (because of prejudice) to figure in the mathematical hall of fame.

The short life of Mrs Kovalevsky (her husband's surname; he was a progressive geologist and friend of Darwin) is like a series of novels with all kinds of cinematic ingredients and romantic content. To date three films have been inspired by her. Of mixed origin, her grandmother was a gypsy and her grandfather a nobleman. She was a descendent of the king of Hungary, Matthias Corvinus. Partly self-taught, up against it because of her sex, a writer, feminist and politician, close friend of Dostoyevsky, Mittag-Leffler and Weierstrass – who did not look kindly upon scientific women, but ended up giving her private classes – Kovalevskaya was the first woman in the world to hold a university chair. She received a Bordin Prize for her studies on the mechanics of the compound pendulum, was the virginal wife to a husband who committed suicide, was politically involved, a beautiful woman who died young due to a cold which turned into flu... plenty to chose from for a drama!

1996 Russian stamp with the face of Sofia Kovalevskaya.

According to legend, it all started when the house was being wallpapered, as in the immense and backward Russia of the tsars, the paper took months to arrive once it had been ordered. The result was that sometimes it was reused in rooms which were less frequently used. One of the children's rooms was wallpapered with paper which was considered second-rate and came from an old stock of notes from

a calculus course with Ostrogradski, and on them the formulae for differential and integral calculation could still be read. Sofia started to try to decipher the formulae by herself, she would gradually start to understand their meaning, like in a wonderful novel. Sofia's father, a man typical of the times, was a lieutenant general with old values to whom women were little more than reproductive machines with negligible minds, was strongly against one of his daughters studying, and even more so if it was something as unsuitable as science, not to mention mathematics.

A friend of the family, Mr Tyrtov had written a book on physics which he presented to them as a gift. Sofia not only read it, but she astonished Tyrtov by her acute comprehension of it. Tyrtov begged the general to let her study this field, but he still refused. In order to gain her freedom, Sofia arranged a 'white' wedding with Vladimir Kovalevsky, a marriage of convenience in which both spouses excluded sex from their relationship. It was the price of freedom. And the rest is history and can easily be followed at other sources; we are not going to dedicate more space to it.

Chapter 6

Recent Mathematicians

A mathematician is a machine for turning coffee into theorems.
Paul Erdös

A somewhat strange deacon

The deacon Charles Lutwidge Dodgson (1832–1898) never managed to become a priest. Instead he is better-known by his *nom de plume*, Lewis Carroll, with which he has gone down in history. The author of, among other works, *Alice's Adventures in Wonderland* and *Through the Looking Glass* was also a mathematics and logic professor with strange hobbies and an academic life that ran in parallel, like Doctor Jekyll and Mr Hyde. Everyone had him down as an exuberant fantasy writer, and his normal existence was that of an amateur photographer who lived off his mathematics classes at Christ Church College, Oxford.

A scene from Alice's Adventures in Wonderland, *illustrated by Sir John Tenniel.*

Carroll's life brims with anecdotes, but perhaps the most famous of them is, unfortunately, not genuine. It is said that Queen Victoria read his works of fiction with great delight, and ordered that the author's upcoming work be purchased and delivered to her. The next book received by the queen was *An Elementary Treatise on Determinants.* Obviously it is not what she expected.

Excellency and its scale

The name of William Thomson (1824-1907) may not mean anything to you. It is his real name, but in books he figures as Lord Kelvin, the title with which he was ennobled in the Victorian era for his efforts towards the laying of the first transatlantic cable, the installation of which did so much to unite America with Europe.

In his youth, Lord Kelvin must have been fairly disliked in his neighbourhood. He was the son of a maths professor, he had been to London (he was Scottish; some might say provincial), he had learnt French in Paris and was so intelligent it was almost insulting. An article of his for the Royal Society of Edinburgh had to be submitted by an older professor because if the scholarly society had known the age of the author (he was still an adolescent), it would have rejected it, assuming it was a joke.

Much is said of him, a lot of which is delectable. For example, when he was a professor it is said that he had three boxes in front of him which he had named 'Purgatory', 'Heaven' and 'Hell'; in each class he would take a name out of Purgatory and make the corresponding student recite the lesson. Depending on their performance, their name would go into one of the other two boxes.

The fact is that his precociousness and the comfortable surroundings in which he lived made him somewhat big-headed, and at Cambridge he received his first slap from the real world. It was a university tradition (until 1909) for third-year mathematics students to sit mathematics exams and subsequently be marked and graded one after another according to the given score. The last place was awarded an imaginary wooden spoon (a custom which persists in rugby), and the first placed students were called wranglers: first wrangler, second wrangler, etc.

Being first wrangler was a great honour; Cayley, Herschell, Littlewood, Eddington, Adams, etc., all achieved this position. There was even a case where the title of first wrangler went to Lord Rayleigh (the Nobel Prize winner) and a note had to be published in the *Times* clarifying that the position was only due to his grades, not

his title and influence. Lord Kelvin passed the exam and, with a certain indifference he sent a servant of his to check the final list. When the servant came back Lord Kelvin did not doubt for a moment that he was the first wrangler and he enquired: "Well, who is the second wrangler?", and the servant responded "You, sir". The first wrangler was someone else. The records do not tell us if the servant remained passive. It is not that second wrangler is bad; for example, James Clerk Maxwell and J. J. Thomson, who discovered the electron, were given that title. Other students who later became famous occupied modest positions on the scale: Hardy was fourth; Bertrand Russell, seventh; Malthus, ninth; Keynes, twelfth. Excellence, as we have seen, cannot be measured by any exam.

William Thomson, 1st Baron Kelvin.

It was not Mittag-Leffler's fault

Alfred Nobel (1833-1896) the explosives manufacturer is responsible for history's most prestigious awards, the so-called Nobel Prizes. There are Nobel Prizes for physics, chemistry, literature, medicine and peace. Later in 1969, the Nobel Foundation came to include economics, an award financed by the Royal Swiss Bank. The obvious question is why is there no Nobel Prize for mathematics? In the 21st century there is a mathematical equivalent to the Nobel Prizes: the Abel Prize, so-called in memory of the great Norwegian mathematician Niels Abel (1802–1829)

and awarded by the Norwegian Academy of Science and Literature. It was awarded for the first time in 2003 and has been received by:

Year	Abel Prize
2003	Jean-Pierre Serre
2004	Michael Atiyah Isadore Singer
2005	Peter Lax
2006	Lennart Carleson
2007	Srinivasa Varadhan
2008	John Thompson Jacques Tits
2009	Mijaíl Grómov
2010	John Tate
2011	John Milnor

It is a little ironic that the Abel Prize comes with an award of 550,000 dollars when Niels Abel literally died of starvation.

There are several explanations for Nobel's actions, but they can be summarised in two examples, and both involve the most important Swedish mathematician of the time, Gösta Mittag-Leffler (1846–1927).

Gösta Mittag-Leffler, the man who was not guilty of anything.

One of the best-known 'explanations' states that Nobel's lover (other conspiracy theories talk of his wife) was snatched from him by Mittag-Leffler, an event that was met with indignation from Nobel, who decided not to establish a reward for mathematicians. It is true that Mittag-Leffler was tall and handsome, but we do not even know if he ever met Nobel, as their interests were very far removed – even physically since Nobel left Sweden in 1865 and only returned to his country on a few occasions. By the way, the story about Nobel's wife can be disproved: Nobel remained single for his whole life.

Other, less translucent 'explanations' attribute Nobel's decision to the mathematician rubbing a few people up the wrong way in the procurement of his personal fortune. Nor is there a grain of truth in this, as Mittag-Leffler was not rich and did not become rich through financial operations he simply married a rich lady.

The conclusion is that Alfred Nobel did not institute a Nobel Prize for mathematics because it did not occur to him to do so; mathematics had nothing to do with his everyday life.

Banned by his wife

The physical or mental abuse of women is unacceptable. But at the dawn of the 20th century this was not yet the case, as Amalie Emmy Noether (1882–1935), perhaps the most important female mathematical mind the world has ever seen, could testify. Her misadventures began early. In 1915, the University of Göttingen in Germany did not accept this female doctorate with insurmountable intellectual prestige as a professor. The argument given by this very prominent institution was one that seems to have been drawn from an anthology of nonsense.

At the height of World War I, the assembly's decision-making minds pharisaically wondered how the hairy-chested soldiers would react upon returning from defending their homeland's holy interests to find that they were being given mathematics classes from an inferior being such as a woman. Possibly they also whispered in each other's ears: a woman teacher meant a vote against them among the faculty's administration. Result: Emmy Noether did not alter the testosterone-fuelled status quo in 1915. A blackball for Emmy. David Hilbert (1862–1943), who was the chairman and had grown tired of such nonsense, claimed in vain: "What does the sex of the candidate matter? The board of directors is not a bath house." Much later, in 1919, Emmy was finally accepted by the university, but not given a specific post or salary.

When, in 1933, she emigrated to the United States due to anti-semitic pressure from the Nazis, she had to face similar problems of intolerence. Her position at Bryn Mawr College was very well paid, but the institution was for girls only and did not boast the status of university. The University of Princeton hired only men. Emmy was not hired by any university and she found herself faced with the somewhat ridiculous paradox, that a mere college professor was giving conferences and seminars in the Institute For Advanced Study at Princeton, where some of her ex-students had offices, and in whose hallways Albert Einstein, for example, could be found.

Hilbert's macabre enthusiasm

What follows only portrays part of the personality of David Hilbert, Poincaré's successor to the title of the world's best mathematician and scatter-brained sage to top all others. Many anecdotes have been told about the state of semi-perpetual cluelessness in which Hilbert lived his mundane existence, but what follows is delectable.

David Hilbert.

Once, one of Hilbert's more notable students submitted a sketch of a demonstration of the famous Riemann hypothesis to him, and Hilbert needed God and his army to find an error in the young man's reasoning. That student really was promising.

However, one day the student passed away, and Hilbert, who was deeply affected by this, asked his family if he could say a few words over his grave in the cemetery. On the day of the ceremony it was raining. There, in the middle of a forest of umbrellas and tears, Hilbert said a few words, while the congregation looked on entranced: no less than the great Hilbert was honouring a student's death with a few words. But Hilbert quickly got excited, he mentioned his death and his virtues, surveyed his interests and his knowledge and continued with the Riemann hypothesis. And to the horror and stupification of the attendees he explained it so that they could understand it. "For example if we take a function $f(z)$ where z belongs to the field of complex numbers..." An elementary course in mathematical analysis followed. Just imagine the scene, before a dead body and a grave, a rainy day, the deceased's stricken parents, relatives... and in front of all of them an enthusiastic speaker giving a course on functions with complex variables. Let's leave it at that.

As you would expect, there have been many scatter-brained mathematicians about whom we could tell stories until the cows come home. On one occasion Witold Hurewicz (1904-1956) parked his car and started work as usual. When he finished he left... but by train; he forgot that he had got there by car. "Well... that's nothing," you might say. But the following day, Hurewicz wanted to go back to his office, he noted with annoyance that he did not have a car and immediately reported its theft to the police.

The ineffable Paul Erdös (1913-1996) was with an acquaintance and he asked him, out of courtesy, where he was from. "From Vancouver," came the answer. Erdös' face lit up: "Ah, then you might know my friend Elliot Mendelson." "I am Elliot Mendelson," he replied. That really is top-class absent mindedness.

Easy and difficult

One of the true greats of economics is John Maynard Keynes (1883-1946), 1st Baron Keynes of Tilton. There is no need for us to sing the praises of someone who is so famous, as this has already been done countless times. Keynes himself once explained that in Berlin, the winner of the Nobel Prize for physics Max Planck (1858-1947), the father of quantum physics and undoubtedly the owner of a visionary mind, had confessed to him that when he was young he wanted to be an economist – but he found economics too difficult. A clear confession of impotence and modesty which Keynes took magnanimously and as a homage to his own talent. But his joy cannot have lasted long. In a meeting that took place a couple of days later at King's College,

Cambridge, Keynes cheerfully told of Planck's confession. One of the attendees, historian Lowes Dickinson, explained that the polymath Bertrand Russell had once told him that, actually, when he was young he wanted to be an economist, but he had abandoned the idea because he thought economics was too easy.

What seemed difficult to a winner of the Nobel Prize for Physics seemed easy to one of the Nobel Prize for Literature. Nobody awarded Keynes with the Nobel Prize for Economics because it had not yet been conceived.

A question of parity

Ukrainian physicist and astronomer George Gamow has told the story of an important conference between many important physicists in 1929. The Klein-Nishina formula was being presented, a formula that is fundamental for the understanding of elementary particles, as it was related to something as significant (to the initiated) as the dispersion of photons in the field of quantum chromodynamics. The article that contained the famous formula was titled *Über die Streuung von Strahlung durch freie Elektronen nach der neuen relativistischen Quantendynamik von Dirac* (On the Dispersion of Radiation by Free Electrons According to Dirac's New Relativistic Quantum Dynamics) and it was the work of Oskar Klein (1894–1977), Swedish physicist, and Yoshio Nishina (1890–1951), one of the revered father figures of modern physics. The conference was given by Nishina himself and among those attending was the genius's silent (but dangerous) guardian, English physicist and mathematician Paul Dirac (1902–1984), who had not yet received his Nobel Prize.

Nishina was chalking his arguments onto the board without any great difficulty, until one of the attendees pointed out to him that his last result contained a minus sign which was not present in the original article. Nishina took it in his stride, he must have made a superfluous error in the change of sign at some point during the labyrinthine path of the calculations. "Have a look for it yourselves, as the article contains the correct change of sign somewhere." Then, and only then, one of Dirac's eyes opened – he had appeared to be asleep during the entire demonstration, as was his habit: "Look for it in an odd number of places," specified Dirac deliciously punctiliously. Indeed, it does not matter if the error is in one sign or in three, in five, in seven, etc., the only thing that matters is the parity of the error, that the number of errors in the sign is odd. Some would say it was Dirac's mathematical spirit talking, others might think it was Dirac's desire to wind people up.

Paul Adrien Maurice Dirac, winner of the Nobel Prize for Physics.

The third in the group

There was a time when the name of Sir Arthur Eddington (1882–1944) was revered by everyone. His status as an astrophysicist would acquire gigantic and universal proportions; he was practically an official sage. One of his manias was numerology, and he made great use of what he called the cosmic number, the equivalent to the number of particles in the Universe, evaluated by Eddington himself at

$$136 \cdot 2^{256} = 15{,}747{,}724{,}136{,}275{,}002{,}577{,}605{,}653{,}961{,}181{,}555{,}468{,}044{,}717{,}914{,}527{,}116{,}709{,}366{,}231{,}425{,}076{,}185{,}631{,}031{,}296,$$

a literally astronomical figure that Eddington handled with elegance. He was a magnificent mathematician, very talkative and had a magnetic personality.

He was supposed to be intelligent enough to understand Einstein's theory of relativity, which was at the time a paradigm of the incomprehensible and mysterious. There is an anecdote about the theorem involving Eddington. The Polish-American physicist, Ludwik Silberstein (1872–1948), a renowned expert in the theory of relativity, complimented the Englishman by saying: "It is said that you are one of only three people in the world capable of mastering Einstein's theory of relativity." Eddington thought a little and replied: "And who is the third?" Of course, in his mind he had already counted himself and Einstein. If Eddington had been modest, nobody could have faulted his wit.

Sir Arthur Eddington (on the right) sitting next to Albert Einstein.

The mathematician who never existed

When looking for the crème de la crème of funny stories about mathematicians, a common recourse is to talk about Nicolas Bourbaki. We have not included his date of birth because, in fact, he does not have one. Nicolas Bourbaki, one of the most influential personalities in 20th-century mathematics, actually does not exist, he is a figment of someone's imagination.

Although it is difficult to put a date on the birth of someone who does not exist, it can be said that more or less in 1935, when the winds of the war that were to shake the world were still mere breezes, a group of young mathematicians, among whom were André Weil (1906–1998), Henri Cartan (1904–2008), Claude Chevalley (1909–1984), Jean Dieudonné (1906–1992) and a few more, agreed to draw up a mathematical treatise on a new approach that would work for everything from the general through to the particular. The texts would be anonymous, the fictitious author would be Bourbaki, and the rules of the game could not be more ideal or more progressive.

Before the final version, each manuscript would circulate around the members of the group ostensibly in order to be improved by personal contributions. What actually happened was that the meetings ended up being shouting matches over the drafts (where the thundering voice of Dieudonné prevailed) but often degenerated into genuine pandemonium. Then the war came, and the post-war period, and

successive, indigestible but unassailable books were published under the name of Bourbaki and their advanced content became legendary, because of their generality, the concept of mathematics that they preached and, it should be said, because of the originality of their collective authorship and anonymity. Little by little, other figures, not all of them French, were added to the group, which was continually renewing itself.

Names such as Laurent Schwartz (1915–2002), Roger Godément (b. 1921), Samuel Eilenberg (1913–1998), Jean-Pierre Serre (b. 1926), Alexander Grothendieck (b. 1928), John Tate (b. 1925), Serge Lang (1927–2005), Alain Connes (b. 1947), Jean-Christophe Yoccoz (b. 1957) are affiliated with the glorious name of Bourbaki along with many others that we have not mentioned – both because it would be a very long list and because it is often unknown whether they belonged to the Bourbaki group or not. Bourbaki, on the other hand, continued to look remarkably young and boasted an unequivocally French sense of humour. Entire books could be written full of anecdotes about Bourbaki, a mythical name which, by the way, comes from a little-known French general.

One of these anecdotes perfectly portrays Nicolas Bourbaki. Ralph Boas (1912-1992), president of the Mathematical Association of America, assumed the duty of clarifying Bourbaki's paper for the general American public in the pages of the *Scientific American*, and there he didactically explained the circumstances of the case – that Bourbaki was a pseudonym for a group of mainly French professionals, etc. Imagine his surprise when, one day, he found a letter in the post from one Mr. Nicolas Bourbaki, in which he energetically protested against his own non-existence.

Of course, Boas took this in good humour, as he assumed it to be a joke from one of the Bourbakists. But this was a mistake, he did not realise the perverse and vindictive imagination of Bourbaki. Shortly, mysterious and widespread rumours began to crop up which advised mathematicians of Boas' non-existence. The rumours had it that it was the name adopted by a group of young American academics, but it was actually no more than a pseudonym for a non-existent entity called Boas...

Finally, let's charitably kill off Bourbaki. In 1968 his obituary appeared, announcing his definitive extinction. At the age of 33, like Jesus Christ, Bourbaki left our world. The obituary, issued by the grieving relatives/authors, announced a funeral service and, in one final gag, it sent a message of condolence to all the Bourbakists and sympathisers citing a verse from the book of an imaginary saint, Grothendieck IV, 22.

A computer and the Cold War

World War II was followed immediately by another, undeclared one, the so-called Cold War, more diluted in time and eventually and definitively won by the West after the fall of the Berlin Wall. In this period of 44 years, the silent competition between the two sides was not only political but also affected the scientific world and even the apparently unrelated field of mathematics.

In 1945, shortly after the end of the armed conflict, the USSR found a world of knowledge for which it was not prepared. For example, there was a great interest in computational technology among its intellectual elite, but there was not even one miserable computer in the country. And when they sought to buy one to see how it worked and copy it, they stumbled upon all sorts of difficulties.

The same year, the US built the ENIAC, the first effective digital computer. The communist bureaucracy, faced with the physical impossibility of obtaining or copying such a giant (and scarce because there was only one) machine, issued a letter from the Soviet government's purchasing commission to the University of Pennsylvania, requesting to purchase a copy of what the USSR called the 'robot calculator'. When the letter was received it was immediately forwarded by the dean of the university to the US military authorities. The letter never received a reply, so the USSR had to get by without an ENIAC. And how they got by! Computational mathematics experienced significant development at the hands of the brains of the Eastern Bloc. They even found time to create computational science geniuses such as Andrei Kolmogorov (1903-1987).

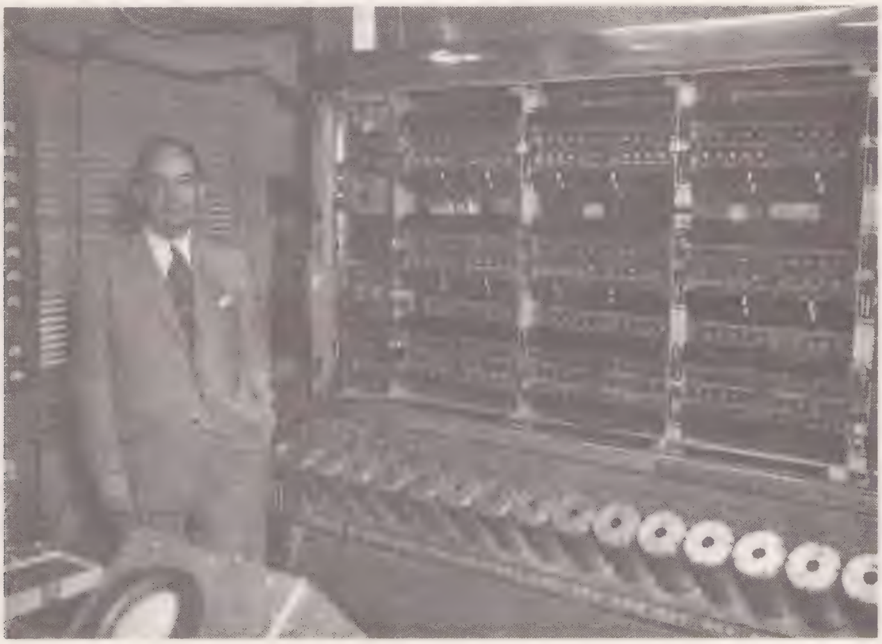
An alien in the United States

John von Neumann (1903–1957), Johnny to his friends, was born a Hungarian and christened János. But his peers were tempted to think that he was actually an extraterrestrial, as his memory, speed and calculating ability, as well as the range of his interests and efficiency of reasoning were truly inhuman. He may have been the last person who was capable of mastering all the mathematics of his time; this would certainly not be possible today as the scope of the field is too broad.

Another mathematician, George Pólya (1887–1985), a man who always stood out for his shrewdness in proposing problems, once commented that a particular theorem had not been proven. Within a few minutes Von Neumann approached a

blackboard, picked up a piece of chalk and demonstrated it. It is said that, since then, Pólya regarded him with a touch of terror.

Hans Bethe (1906-2005), winner of the 1967 Nobel Prize for Physics, classified the problems presented at mathematical seminars into ten levels of increasing difficulty: "A level 1 problem is the type that even my mother can understand. Level 2s can be understood by my wife, let's say..." Skipping forward to save time: "Level 7 is formed by any problems which I can understand. Level 8 would be constituted by those statements which only the lecturer and Johnny von Neumann are capable of comprehending. Level 9 contains those which only Johnny would understand, as at this point the lecturer would not understand a thing. And level 10 would be the set of problems which not even Von Neumann would understand. But the truth is, there are very few of those left."



John von Neumann was one of the founding fathers of electronic computing.

Norbert Wiener

US mathematician Norbert Wiener (1894-1964) is very well known for being the creator of cybernetics, as well as for being a famous child prodigy. He has passed into popular anthology as the protagonist of so many stories of his well-known scatter-brained wisdom that we are not going to include them here to avoid overkill. However, there is a lesser-known example which is worth the time. Imagine a

class in the legendary Massachusetts Institute of Technology where Wiener imparted his wisdom at top speed, flooding the board with symbols and ideas and sailing, to the dismay of his audience, through a dazzling ocean of concepts and theorems into which students would inevitably sink. One of the students decided to ask for a ceasefire in the bombardment: "Please could you repeat it all a little slower?". Wiener kindly complied and did what was asked of him, but in the way which he understood the request. The problem was that he was going too fast? OK, relax. Wiener stood smiling at one side of the board and stayed still and silent for several minutes. After an interval of time, which he considered sufficient for mental digestion, he returned, still smiling, to the blackboard, drew one final energetic full stop in chalk at the end of everything else he had written, and the class finished there. And, of course, nobody understood a thing.

An illogical constitution

The logician who has had the greatest impact on contemporary mathematics is Austrian-American Kurt Gödel (1906-1978), a great mathematician who will figure in all future scientific encyclopaedias for his professional achievements and, unfortunately, also in all collections of anecdotes due to his strange character, which became more accentuated as the years passed.

In his final years, Gödel considered it convenient to adopt US nationality and, according to the customs of his adopted country, he went through the procedures necessary to do so, which require, among other things, having a couple of sponsors and swearing allegiance to the constitution before a judge. The sponsors were two friends of his, and what friends they were! They had both graduated, like Gödel, from the Institute for Advanced Study at Princeton. One of them was Albert Einstein, and the other, Oskar Morgenstern (1902–1977), the economist who created (together with John von Neumann) game theory. Both of them feared Gödel's sense of reason during the ceremony, as they knew of his progressive paranoia and that, although he wanted to obtain his new citizenship, he was going to make things difficult for himself. He had read the US Constitution, and his sharp logician's mind had found what he considered to be grey areas in its text. Specifically, Gödel alleged that the consitution had errors in it and that it included loopholes through which a dictatorial regime could be smuggled.

The time of judgement came and the judge began to talk, taking a seat with such distinguished characters – no less than three of the most powerful minds on

the globe. The judge reminded Gödel with great courtesy that what had happened in his homeland (which he unfortunately thought was Germany, but Gödel was actuallyan Austrian) would never happen again: "The American Constitution will not allow a dictatorship in our country." This was like a red rag to a bull. Gödel went off on an incendiary speech. According to him, a loophole in the constitution authorised dictatorship. His sponsors swiftly interrupted his speech, perhaps by mentioning the weather. In the end Gödel obtained the nationality he was after; the judge granted it to him, presumably so that he did not have to listen to him any more.

"All's well that ends well," Einstein must have thought. Morgenstern probably thought "How did I get involved in this mess?" while Gödel probably complained "They didn't let me explain it!"

Kurt Gödel at the Institute for Advanced Study, Princeton.

A special vocabulary

Paul Erdös was not only a prolific and special mathematician, he also expressed himself in a special language when he was not doing mathematics. His particular way of expressing himself, a consequence of his obsessive practice of mathematics, would justify several pages, but there is no need to go into such detail. A few select examples will do. They can be found easily online:

Term used by Erdös	Meaning
Boss	Woman
Slave	Husband
Epsilon	Child
Captured	Married
Recaptured	Remarried
Liberated	Divorced
Poison	Alcohol
Noise	Music
To preach	To give a class
Supreme fascist	God
To arrive	To be born
To exist	To do mathematics
To die	To abandon mathematics
To leave us	To die
Oral exam	Torture
János	Hungary (from János Kadar, Hungarian communist dictator)
Samland (from 'Uncle Sam')	USA
Joedom (from Josef Stalin)	USSR
On the long wavelength	Communist (from the colour red)
On the short wavelength	Fascist (blue)
Sam and Joe show	International news
The Book	A collection of the most brilliant mathematical demonstrations
Trivial entity	Someone who does not do mathematics
My mind is open	I am ready to do mathematics
What was it when it was alive?	What kind of meat is that?

A perfect con

Mathematicians are capable of inventing such wonderful cons, it is a shame that it is not their job. The mathematician John Allen Paulos (b. 1945) has triumphed in the world of literature, writing mathematics books which, by the way, have sold very well internationally.

The most successful of them is titled *Innumeracy*. In it, Paulos complains of modern man's ignorance when it comes to the everyday numerical world. For example, there are millions of people who do not correctly use percentages, and it is deplorable that even in newspaper headlines such a simple concept is misused.

However, this is not why we are talking about Paulos. In *Innumeracy* an investment con which anyone with a certain initial capital could undertake is explained in detail. We include it here at the risk of being detained by the authorities.

Let's suppose that we send 64,000 messages with a special content. In half of the messages we recommend a particular investment and in the other half we recommend not investing. There will be 32,000 messages which turn out to be good advice, a good number. Now we repeat the operation, let's say 5 more times, each time rejecting the addresses of the messages in which we failed. In the end we will have 1,000 addresses of people who will have received six consecutive messages recommending a good investment.

In such an insecure and competitive world, six correct messages in a row is quite incredible. OK, now we have 1,000 potential subjects for our con. We can almost certainly persuade someone that if one of our 1,000 subjects hands over an amount we will invest it correctly. And, of course, we will not return the money to our gullible victim. Let's make it clear that it was Paulos who said it, not us.

You cannot trust *The Da Vinci Code*

The novel *The Da Vinci Code* was not only a best-seller in every language, but also a great read for mathematical spirits, given that many of the puzzles are arithmetic or geometric. The novel somewhat cruelly criticises institutions such as the Opus Dei, which has won it a few detractors. These detractors could find more amunition for their acrimony if they realised that even some of the mathematics in the plot in *The Da Vinci Code* are inaccurate. We are going to give an example.

In Chapter 22, evil hair-shirted and somewhat stupid Silas is entranced by the pink line in the Saint Sulpice church. The line is a metal band that runs through the floor and was constructed in 1727 by architects as a gnomon, that is, an astronomical device to make the shadow created by the sunlight indicate the exact time of the summer solstice. The problem is that the author, Dan Brown, identifies the line on the floor as the line that indicates the Paris meridian. But this is not the case. The authentic meridian is located on an imaginary line quite a few

metres from there. It can be seen in the Paris Observatory as a continuous line and on the ground of the city it is indicated by more than one hundred bronze medallions bearing the name Arago, the first mathematician to calculate the position of the meridian. The first medallion is in the centre of the famous pyramid of the Louvre.

The pink line on the floor of the Saint Sulpice church, which stretches as far as the obelisk in the background (source: PHGCOM).

The metal band which indicates the meridian in Paris in the city's observatory.

A genius in action

Englishman Stephen Hawking (b. 1942) has moved from the precise field of science to that of mass media and the gossip columns. An eminent scientist with extraordinary abilities, the prisoner of a wheelchair and slave to a degenerative incurable disease, he is, undoubtedly, fodder for the press, especially the tabloids. If we add to this that Hawking's main field of work is astrophysics, we have a strong candidate for occupying the position of exemplary scientific genius left open by Albert Einstein.

In his youth, Hawking studied at Oxford, where he was mercilessly subjected not only to lectures, but also to the frightfully tough problems the students had to solve between classes in order to demonstrate that they had understood everything.

On one occasion, he and his friends were faced with a set of particularly difficult problems and some set about solving them, even through the night. The next day, Hawking's classmates had solved two and a half problems between them. Our hero put his mind to the task after breakfast, with just three hours to go until the class where the homework had to be handed in. Just before class Hawking appeared, looking somewhat dejected. "Come on, Hawking, how did it go? Did you solve any?" Answer: "Damn, I wasn't able to finish them. I only finished the first ten."

Dumb, but not that dumb

Jean Leray (1906–1998) was one of the most important French mathematicians of the 20th century and, although he was closely associated with the ideas of the Bourbaki group, he did not form part of the now legendary collective. Leray was a patriot and, being a man of recognised intellect, he was categorised as a danger by the Nazis occupying France. From 1940 to 1945 he was held in the prisoner of war camp in Edelbach.

Leray was an authority on fluid mechanics. His results from the study of one problem – the Navier-Stokes equations – were fundamental to solving it. Fearing that the Nazis would find out and force him to use his knowledge for whatever military purposes suited the needs of their war, Leray radically changed his field of interest and dedicated himself to topology, a branch of mathematics which, at the time, was reputed to be useless and of completely no interest to the military. Leray being Leray, he could not help becoming, despite being in a prisoner of war camp, a world authority on algebraic topology.

Nothing lasts forever, though; the war ended and Leray survived and was freed. When he returned to work he left topology, his companion for so many years, behind and returned to the land of equations in partial derivatives. He had not lost his touch.

More than you bargained for...

There are certain types of humour which you have to admit are annoying. There is a wonderful example on page 75 (in the 1965 edition) of the book *Méthodes mathématiques pour les sciences physiques,* by Laurent Schwartz (1915-2002), a recognised member of the Bourbaki group and winner of a Fields Medal in 1950. The page contains a collection of end-of-chapter problems. The humorous element can be found in problem 8, the wording of which, translated from French, is as follows: "One

of the previous questions is false. Which one?" Any poor student who has struggled through problems 1 to 7 will undoubtedly curse here, instead of chuckling at the author's sense of humour. Sweating blood to solve seven problems is one thing, but to later be told that one of them is false, but not which one, is quite another.

But thinking about it, is our revered mathematics not the art of thinking? Maybe, then, Schwartz hits the nail on the head and problem 8 is a fantastic problem.

Exercices sur le chapitre 1

On dit que les deux normes sont équivalentes si chacune d'elles est plus fine que l'autre.
Montrer que sur $\mathcal{E}^0_{[0,1]}$, $\|\cdot\|_2$ est plus fine que $\|\cdot\|_1$.
En utilisant la suite de fonctions $f_n(t)$ définie sur $[0, 1]$ par

$$f_n(t) = \begin{cases} 0 & \text{pour} \quad t \geqslant 1/n \\ -nt + 1 & \text{pour} \quad 0 \leqslant t \leqslant 1/n \end{cases}$$

montrer que $\|\cdot\|_1$ et $\|\cdot\|_2$ ne sont pas équivalentes

6° Montrer que pour $\|\cdot\|_2$, $\mathcal{E}^0_{[0,1]}$ est un espace de Banach

7° On considère la suite de fonctions $f_n(t)$ définie sur $[0, 1]$ par :

$$f_n(t) = \begin{cases} 0 & \text{pour} \quad t \leqslant 1/2 \\ n(t - 1/2) & \text{pour} \quad 1/2 \leqslant t \leqslant (1/2 + 1/n) \\ 1 & \text{pour} \quad t \geqslant (1/2 + 1/n) \end{cases}$$

Montrer que pour $\|\cdot\|_1$, c'est une suite de Cauchy qui ne converge pas vers une fonction f de $\mathcal{E}^0_{[0,1]}$. Pour $\|\cdot\|_1$, $\mathcal{E}^0_{[0,1]}$ n'est donc pas complet.

8° Une des questions ci-dessus est fausse. Laquelle?

The page from the book Méthodes mathématiques pour les sciences physiques *on which it is revealed that one of the seven problems which have just been proposed is false.*

The merit of Jacques Tits

Tits is perhaps a somewhat inappropriate surname for a prestigious algebraist. It is not easy to imagine a university professor having such an unappealing surname called out in the hallways and in meetings full of English-speaking students. However, the fact is that Jacques Tits (b. 1930), winner of a Wolf Prize (1993) and an Abel Prize (2008), has two significant characteristics. Firstly, he is not English, he is a Belgian who changed his nationality to French when he was asked to give classes at the Collège de France. Secondly, he is one of those experts who demand respect because

of their millions of creative neurons. He is a great specialist in group theory. In fact, there is a simple group that bears his name – the group ${}^2F_4(2)'$, or the Tits group, has 17,971,200 elements. He has also constructed new concepts such as 'building', (in French *immeuble*), which is a combinatorial structure that a group acts on.

In 1995 he was awarded the Croix de Grand Officier from the German order Pour le Mérite. This is not unusual in itself, as Tits is a great scientist and, also, he taught for nine years in Berlin. Things start to get weird when you realise that everything is in French. The order was created in 1740 by Frederick II (the Great), once king of Prussia (French was the official language of the court back then), and only as recognition for acts of war which were considered sufficiently gallant at the time. In around 1842, Frederick William IV created a civil branch of the order which could include non-military people. Alfred Einstein, for example, was one of the people decorated, but the Nazis forced him to return the medal. Since 1952 it has been granted, with great restraint, by the German chancellor.

And do you know who received it as far back as 1918? One Herr Hermann Göring, hero of early German aviation and future *Reichsmarschall.* I do not think either the Belgians or the French have great memories of Göring, but Tits did not mind figuring in the same order Pour le Mérite as Göring; perhaps he even smiled. We cannot ask Göring about it.

I am getting old

This is an inevitable circumstance and they are words or thoughts which have passed through the lips or minds of many people. For some of these people, who have lived a brilliant life, they are particularly painful. Fields Medals, for example, do not award a life's mathematical career (this job is reserved for the Abel Prizes), but mathematical excellence before 40. Sir Andrew Wiles did not receive the coveted medal, even having demonstrated Fermat's famous theorem. When he completed his achievement he was still not 40 years old, but he spent a certain amount of time trying to correct an error in the demonstration, and when he managed it he had already surpassed the fateful age limit by just a few months!

That mathematicians, when they reach 40, go to the elephants' graveyard is debatable; in fact, there is abundant proof that this is not the case, although it is true that a young body appears to bring with it a young spirit. Some professionals, though, notice age differently to others. Laurent Schwartz (1915-2002), member of the Russell Tribunal, father of the theory of distributions and winner of a Fields

Medal, realised that he was getting old when he had to take notes. Schwartz was something like Borges' character Funes the Memorious. He lived nearly all his life at the forefront of science, without taking a single note. He attended conferences and seminars, storing thousands and thousands of blackboards full of formulae in his privileged memory. When he realised that he had to use notes, he himself admitted that he was entering old age. He could no longer remember everything.

Laurent Schwartz was, as well as a mathematician and man of politics, a great collector of butterflies, as proud of his specimens as he was of his beloved mathematics.

Chapter 7

Mathematical Swan Songs

Mathematics is an exact science;
you always know you are going to fail it.
Anonymous

Occasionally a mathematical curiosity cannot be attributed to one specific person, but to various. For example, the death of famous mathematicians is a matter of certain interest; dying of old age in bed is not the same as being executed. Euler himself (whose death will be recounted in this chapter) was a man with experience of Russia because he lived there for a long time. He used to reply to those who accused him of speaking too little: "Did you know that I come from a country where they hang you for speaking too much?" There are stories that carry certain symphonic overtones which affect a lot of people. Let's take a walk among them, we will be wiser for it.

The bells sound like death

We find death strangely fascinating, especially when it occurs in an unusual way. Many a mathematician's journey to the other side has not been free from gruesome details; for example, the great Archimedes (c. 287 BC–c. 212 BC) was pierced by the sword of an over-excited soldier who was provoked by the sage, while he was drawing diagrams in the dust on the ground. Archimedes had answered his threats with something like "Don't bother me now, for God's sake, and be careful not to stand on my drawings." It is an end that was sufficiently dramatic to go down in history, above all because Plutarch relates it, providing it with a certain aura of authenticity.

The death of Eratosthenes of Cyrene (276 BC-194 BC), the mathematician who made the first serious geometric calculation of the diameter of the Earth, belongs to a slightly less fluid part of the truth. It is said that at the age of 80 (which was a great age at the time), tired of the world and blind, he let himself die of hunger.

The death of Evangelista Torricelli (1608–1647) is normally attributed to something as common in his time as typhoid fever. However, a more truculent (and probably false) version attributes it to an irresistible and fatal feeling of shame. Shortly before passing away he was accused by Gilles de Roberval (1602–1675) of plagiarism as a result of some discoveries about the cycloid, and according to legend he perished because of the consternation that such public discredit caused him. It does not seem particularly believable that Torricelli was such a sensitive soul.

The great Leonhard Euler met with a model death, organised, clean and worthy of the world's greatest mathematician, a title that was certainly credited to him his during his life. One morning in 1783, as he often did, he was giving a class to some of his grandchildren, as the schools were not what they are today and many people of means took classes at home. Afterwards, while he was drinking a mug of tea, he mumbled "I'm dying..." and he died. That is it.

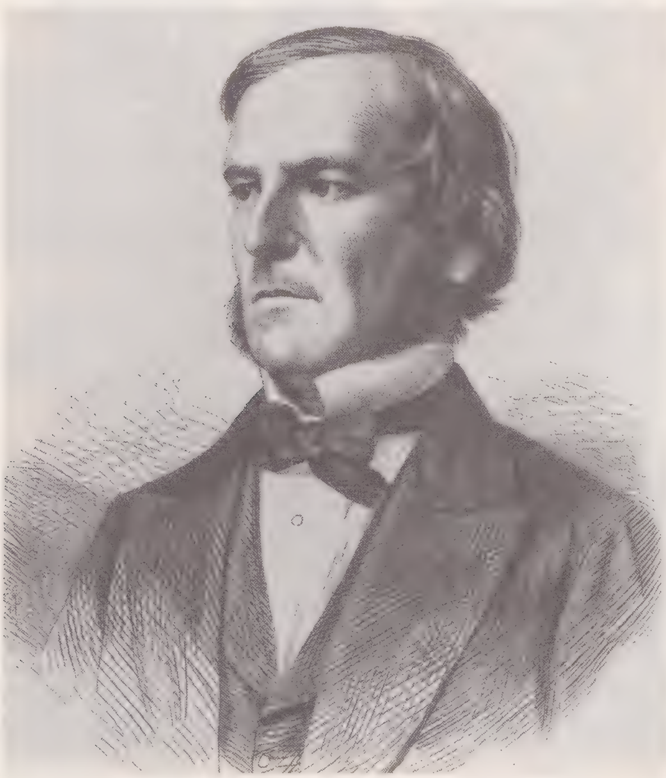
Nicolas de Condorcet (1743-1794), Marquis of Condorcet, appears in the encyclopaedias as a French philosopher, scientist, mathematician, politician and political scientist, and he certainly was all of those things. He was an academic who signed a multitude of mathematical articles in the famous *Encyclopédie Française* and he was a highly regarded analyst. He was also an aristocrat and, by definition, therefore a suspect during the revolution, regardless, even, of his ideas. In the legislative assembly he formed part of the Girondists and was responsible for education in the entire country – a great honour. But when Robespierre and his henchmen arrived, none of this was of any use to him and he had to take to his heels. It is said that he was recognised and imprisoned because he ordered an omelette and with his lack of real-life experience (he had always been served by servants), he naively asked the caterer to make it with 12 eggs. His attitude was interpreted by the caterer as that of a typical aristocrat and he reported the disguised Condorcet. So, he died a few days later in prison (not from the omelette) and it is suspected that, knowing how Robespierre treated prisoners, he committed suicide.

Little known, perhaps because of his lack of drama and his bourgeois affiliation, is the death of Joseph Fourier (1768-1830), a man who dedicated so much effort to analytically representing the theory of heat. It is said that when he was young he used to write sermons for preachers in his parish, such was his precociousness and talent. Fourier suffered from a genuine obsession with high temperatures, and at home he breathed (with great difficult) in an asphyxiating atmosphere. He picked up his obsession in Egypt, where he was travelling with Napoleon army, and was convinced that heat conserved everything, including people's lives. He had seen the

rigorous heat of the desert preserve bodies from decay (it is actually the aridity) and returned convinced that heat would conserve his. Fourier used to cover himself heavily in layers of clothing, even in the summer. He was a crusader for heat and nobody could bring him to his senses. And it was heat which, one day, brought his very existence to an end, according to some sources, in the form of a heart attack.

Even more dramatic was the death of Évariste Galois (1811-1832), who passed away at the age of 20 as the result of a duel over love. To add a strange twist to the story we should add that Galois used the hours preceding the duel writing up a summary of his mathematical ideas for a friend of his, asking him to make sure they were sent to Jacobi or Gauss, minds who, in his judgement, were capable of appreciating them.

It is said that George Boole (1815-1864), the magnificent logic specialist and professor in Cork, Ireland, also met his end through another misguided healing practice. One unfortunate rainy day, Boole got soaked and consequently died of pneumonia. His wife lovingly cared for him, but in the Irish style of his time and following, it seems, that ignorance prevailed in that period, as the anti-homeopathic treatment, as we will call it, involved pouring bowl after bowl of cold water over his body. Poor Boole brought an end to his torture when he died.



Pneumonia took the life of George Boole.

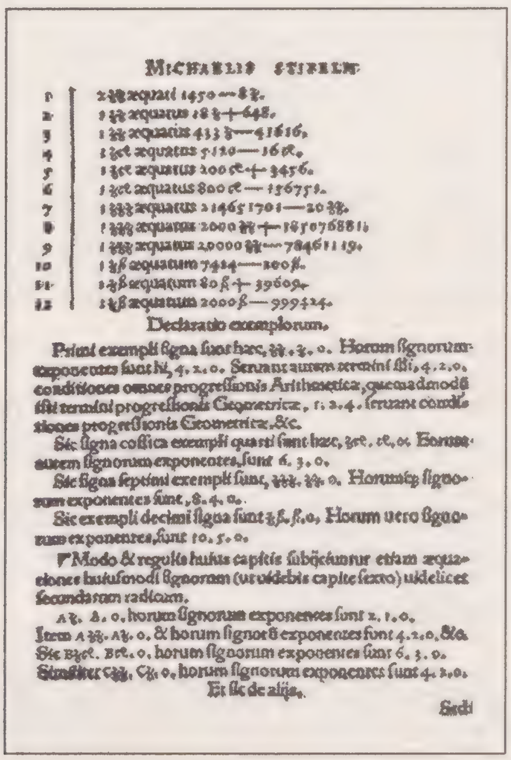
The talented Englishman Alan Turing (1912–1954), a perfect athlete, long-distance runner (he got around by running between his favourite libraries), one-time assistant to Von Neumann and creator of a large part of modern computational methods, found posthumous fame among the general public by managing the intellectual army at Bletchley Park who were responsible for deciphering the Nazis' naval codes and for ensuring an Allied victory in the Battle of the Atlantic. The problem was that Turing was homosexual, something that was considered sinful, abnormal and perverse at the time, and susceptible to be treated, in the best of cases, as an illness. Once his superiors became aware of his 'illness', he accepted the treatment and suffered from depression, during which he dedicated his time to the study of new chemical substances which he recklessly tested on himself. Maybe it was an unfortunate coincidence or perhaps he thought that the definitive solution to his problem was to be found in death, but the fact is that he took a bite of a poisoned apple and died. Parallels with *Snow White*, the classic animated film created in 1937, are impossible to ignore.

Slate statue of Alan Turing kept in the National Museum of Computing at Bletchley Park.

An equally horrific way of dying was that of Kurt Gödel (1906-1978), the logician to whom we owe the incompleteness theorems that changed current thinking about the foundations of mathematics. Gödel was always extravagant and somewhat paranoid, something that worsened with time. In his final years, by then mentally unstable, he was convinced that people wanted to poison him (a common belief among those who suffer from persecution complexes) and only accepted the food offered to him by his devoted wife Adele. Unfortunately she became ill and had to be hospitalised, which left Gödel without a source of food. He lost weight until he was reduced to an unbelievable 30 kg and starved to death.

Horoscopes and predictions

We all have a pending debt with the Lutheran Augustinian monk Michael Stifel (1487-1567). He invented something very similar to logarithmic tables completely independently of Napier. His masterpiece, *Arithmetica integra*, used the symbols +, – and $\sqrt{\ }$, which is why it holds such an important place in the history of algebraic notation.


MICHAELIS STIFELII

Declaratio exemplorum.

Et sic de alijs.

A page of Arithmetica integra *in which the use of the symbols + and – can be seen.*

However, it was not his most popular work or the piece which concerns us here. In *Ein Rechenbuchlin vom Endchrist. Apocalyps in Apocalypsim* (*A Book of Arithmetic about the Antichrist. A Revelation in the Revelation*), Stifel created a horoscope which predicted, among other things, the end of the world on a specific day in 1533. The date drew nearer and finally arrived and the world did not end. Some writers talk of Stifel's voluntary confinement in a prison, having fled from those who wanted explanations, who, it would seem, were quite justified. In the same pamphlet Stifel plays an ingenious numbers game with the name of the reigning Pope, Leo Decimus (Leo X), and the number of the devil (666) from St. John's Revelation, with which it cannot be said that the Pope in Rome was particularly pleased. In the end Stifel did continue with his successful practice of arithmetic, but he left horoscopes to one side.

At the time, horoscopes enjoyed an irresistible popularity and attraction and Gerolamo Cardano (1501–1576) also succumbed to a similar fate. Cardano had already put his foot in it once with a horoscope for his leader, Edward VI, who was sick with smallpox. He predicted that he would live on in happiness, only for the king to die the following year from tuberculosis. In his old age, Cardano did a horoscope for none other that Jesus Christ himself. He must have thought, quite naively, that he would be fine because it was not the first horoscope he had written. The Church was horrified with Cardano's creation and he felt the full force of ecclesiastical wrath. Although Cardano's life was spared he never wrote about anything related to Jesus again, or to anyone for that matter because he was banned from writing altogether. Despite what happened, his hunger for prophecies was not satisfied. It is rumoured (presumably incorrectly) that he further complicated matters by doing his own horoscope. His life did indeed end on the day he predicted, but that was due to the fact that when the fatal day predicted by his horoscope arrived, Cardano killed himself. Looking on the bright side, his prediction came true and he did not make a fool of himself.

The well-known Abraham de Moivre (1667–1754) also justified his reputation as a prophetic mathematician. Mathematics recognises him as the author of the following equality of complex numbers:

$$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha.$$

Legend also attributes the following to him. In his old age he realised that his sleep was getting increasingly deeper and that every day he slept a quarter of an hour more than the previous one. For a mathematician like him it was nothing short of

rudimentary to calculate the date of his death – or we could say eternal sleep. On a certain calculable date De Moivre would sleep for 24 hours a day and he would never wake up. On 27 November, 1754, De Moivre's prediction came true.

The father of calculus, the great Isaac Newton (1642-1727) was no stranger to the charms of prophecies. Although in his era burning heretics alive and torturing them was beginning to be frowned upon, the fact is that the great man was secretly living in sin. He was a Monophysite heretic, or, if you prefer, Arian heretic, someone who did not accept the divine nature of Jesus Christ. To our modern mentality such things may seem far removed from current concerns, but in times when the Christian dogma was universal, being a heretic was unthinkable and punishable.

The fact is that, as part of his religious research, which the heretic Newton himself considered very important, he spent time predicting the date of the end of the world, interpreting passages of the Bible literally. If you want to confirm this you should approach the Hebrew University of Jerusalem, where the original manuscripts are kept. For his calculations, Newton mainly focused on sections of the book of Daniel and, particularly, chapter 7.

His prediction was that the end of the world would fall 1,260 years from the re-foundation of the Holy Roman Empire, an event which, as we know, was led by Charlemagne in the year 800. Calculating the corresponding sum puts the end of the world at 2060. This does not necessarily mean we will be victim of all kinds of misfortune in 2060, as it is possible that the end of the world is actually benign and consists of the second coming of Jesus, and a kind of global terrestrial paradise will be established. This is not very likely given the way things are going, but still, if Newton predicted it...

Simultaneous discoveries

When many minds are committed to solving the same problem, the phenomenon of simultaneous discovery becomes a distinct possibility. The problem is that the wise men involved, who are often genuine prima donnas, do not like it one bit.

In the, let's say, heroic period of mathematics there were practically no monographic magazines or periodic publications; news was transmitted by means of letters, and was passed from one person to the next like an oil stain slowly expanding on a piece of paper. A long time could pass until it was known that so-and-so, perhaps the resident of a far-off country, had arrived at the correct conclusion to a particular problem. Among the best-known cases of coincidence are the following:

— Decimal fractions were used almost simultaneously by the German Bartholomaeus Pitiscus (1561-1613) in 1608-1612, by Kepler in 1616 and by Napier in 1616-1617.

— Logarithms, embraced in their time as a calculus tool that was nothing short of miraculous, are attributed to Napier (in 1614), but were actually also introduced by the Swiss Jost Bürgi (1552-1632) in 1620.

— The inverse square law, fundamental in physics, astronomy, electromagnetism, etc., was discovered independently by colleagues, who, fortunately, got on well: Newton deduced it in 1666 and Edmond Halley, in 1684. The gap in time is due to one of Newton's many oddities, he did not tend to publish his results. He produced *Philosophiæ Naturalis Principia Mathematica* upon Halley's insistence. Other thinkers, such as Robert Hooke (1635-1703), could probably also figure alongside Halley on the list of discoverers of this law.

— Perhaps the most famous coincidence is the advent of infinitesimal calculus, which unleashed raging controversy, accompanied by understandable, but unjustified, patriotic overtones. History later made sure that the merit was equally divided between the Englishman Newton and the German Leibniz, attributing a common and independent discovery to them both.

— The principle of least squares was discovered almost simultaneously by Legendre (in 1806) and Gauss (in 1809).

— Non-Euclidean geometry is a fundamental part of our cultural heritage, and it was clarified by contributions from Gauss (1829), who kept his results secret, hidden in a box, the Hungarian János Bolyai (1802–1860) in the period from 1826 to 1833 and the Russian Nicolai Lobachevsky (1792–1856), who developed hyperbolic geometry in 1836–1840.

— The theory of duality of projective geometry was stated by Jean-Victor Poncelet (1788-1867) and by Joseph Gergonne (1771–1859), both in 1838.

— Vectors, universally attributed to Hermann Grassmann (1809–1877) were also described by William Rowan Hamilton (1805–1865) in the same year, 1843.

— In 1846 a new planet was discovered in our solar system – Neptune. The discovery was the practically simultaneous work of Englishman Adams and Frenchman Le Verrier. English Astronomer Royal George Biddell Airy (1801–1892), having already caused a scene, completely discredited Adams, but the facts gave him the recognition he deserved in the end. The incident reawoke the worst rivalries between Britain and the Continent and the supposed intellectuals are not shown in very good light by history's impartial judgement. Neptune was found in 1846, but the person to find it first, on paper and based on calculations, was Adams. But his achievement takes nothing away from Le Verrier.

— The theorem of prime numbers was formulated by Gauss, but he never proved it. It is not surprising because the demonstration was far from easy. It was not until 1896 that the Frenchman Jacques Hadamard (1865-1963) and the Belgian Charles de la Vallée Poussin (1866-1962) found proof, each of them independently.

— The Italian Ennio de Giorgi (1928-1996) and US Nobel Prize winner John Nash (b. 1928) solved the 19th Hilbert problem almost simultaneously in 1956. The event had highly negative repercussions on Nash's mental illness, as De Giorgi's demonstration was clearly the first, although the difference was only a few months.

The obvious, which perhaps isn't so obvious

Nobody has ever provided a precise definition of what it obvious or trivial; at least of what is obvious to a given professional mathematician. Generally, 'that is obvious' means 'that is obvious to me', which is not exactly the same thing A joke among students at Princeton said that when Alonzo Church (1903–1995) decreed that something was obvious, it was so obvious that everyone had noticed it; when it was Solomon Lefschetz (1884–1972) who said it, it was definitely false, and when Hermann Weyl (1885–1955) said it, you had to be Von Neumann to prove it. In a way this defines Von Neumann, of whom Peter Lax (b. 1926), an Abel Prize winner, no less, stated: "Most mathematicians prove what they can, while Von Nuemann proves what he wants."

It is said that once, with Von Neumann at the board, a person in the audience raised their hand to ask: "Mr Von Neumann, could that be demonstrated in another

way?" He lowered his arms, looked at the blackboard, thought for a few seconds, turned to his class and said: "Yes". And continued with his lesson.

A similar story is attributed to Godfrey Harold Hardy (1877–1947), who it seems uttered the fateful words "That is obvious," and straight away realised that it was not. He stared at the board for a while, and immediately turned on his heels and quietly left the class, something which was met with murmurs of surprise from his students. Five minutes passed and he came back. "Indeed, it is trivial," were his first words. He turned to the board again and continued with the class. Others attribute the anecdote to Hilbert.

Sometimes obviousness is justified; for example, Alexander Grothendieck (b. 1928), *enfant terrible* of French mathematics and a truly privileged mind, published an article in 1969 entitled 'Hodge's General Conjecture is False for Trivial Reasons'. And it was true, Hodge's conjecture was badly conceived and it had to be reformulated. And given that it has yet to be proven by anyone, there is little hope that it will be proven in the next 100 years, at least. The current formulation of the conjecture is one of the seven millenium problems proposed by the Clay Foundation, which offers $1,000,000 for their solution.

The Ig Nobel Prizes

The Ig Nobel Prizes, which you can guess from the name are a light-hearted imitation of the Nobel Prizes, are awarded every year. The prizes are given, instead of by an academy or a respectable foundation, by a somewhat informal magazine known as *The Annals of Improbable Research*, which scours research articles published around the world for those which are notable for the absurdity of their conception, their outrageous title or their apparently grotesque purpose. And we say 'apparently' because they are very serious articles that contain scientific information conceived for a good purpose, as bizarre as it may at first seem. They are awarded every year in a ceremony which is usually attended by many genuine Nobel Prize winners and to which the prize winners are invited (and generally attend). One or two winners have disappeared from the scientific world in an attempt to escape, but they are in the minority.

Out of interest, there is at least one Nobel prize winner who has also won an Ig Nobel Prize – the Russian-Dutch André Geim (b. 1958) received the Nobel Prize for the invention of graphene and the Ig Nobel for a study on the levitation of frogs subjected to superconductivity.

But an example is worth a thousand words, so we will cite a few prizes which involve mathematics:

— In 1993 the prize was awarded for an article on mathematical statistics by Robert Faid (United States) which was titled *Gorbachev! Has the Real Antichrist Come?* The probability in the article of Gorbachev actually being the Antichrist was estimated at:

$$\frac{1}{710{,}609{,}175{,}188{,}282{,}000}.$$

— In 1994 it was awarded to the *Southern Baptist Church of Alabama* for its estimation, carried out in the United States, state by state, of the number of citizens who would go to hell if they did not repent immediately.

— In 2000, in the computational science section, Chris Niswander (United States) received the award for inventing the software *PawSense* which can detect when a cat steps on a PC keyboard.

— In 2002 it was the turn of Indians K.P. Sreekumar and G. Nirmala for their brilliant study *Estimation of Total Surface Area in Indian Elephants (Elephas maximus indicus)*, clearly an indispensable calculation for unexplained purposes.

— In 2006 a prize was awarded to Australians Nic Svenson and Piers Barnes, who calculated the minimum number of shots needed in order to ensure that nobody in the group has their eyes closed in a photograph. The article was titled 'Blink-Free Photos, Guaranteed'.

— In 2007 the prize for physics was awarded to L. Mahadevan (United States) and Enrique Cerda Villablanca (Chile) for the article *Geometry and Physics of Wrinkling*, which speculated about the common phenomena of creased paper, a nightmare, for example, when working with children.

— In 2008 a distinction went to Gideon Gono, the governor of the Reserve Bank of Zimbabwe, no less, an institution with little prestige in our times, who published a notice, to aid and educate the clients and users, in which the

denominations of the notes issued by the bank were explained. They ranged from 0.1 to 1,000,000,000,000,000 dollars, figures between which there must have been at least a few notes of legal tender. It seems that in the domain of Robert Mugabe there is quite a bit of inflation and it is explained that not everyone in the world knew how to say "Give me 1,000,000,000,000,000 dollars of cornmeal, please." Imagine the food crisis that might be caused if the vendor understood "Give me 100,000,000,000,000 dollars of cornmeal, please." That a simple arithmetic-linguistic error can cause death by starvation is worthy of any prize.

Mathematicians should go to jail

OK, not all of them, but at least some of them have deserved to. We are talking about cases in which the imprisonment was clearly due to circumstances unrelated to mathematics. Bertrand Russell, for example, was sent to jail for pacifism during the First World War; Casanova languished in several dungeons for political reasons; American mathematician Ted Kaczynski (b. 1942), better-known as the *Unabomber*, was jailed for terrorism. We are going to cite a few, less straightforward cases:

— French mathematician André Weil (1906–1998) is almost better known for being the younger brother of philosopher Simone Weil than for being one of the 20th century's most important scientists. And what is not mentioned is that, for a long time, Simone had a complex about her brother's precociousness and intellectual gifts. It is also relatively little known that he spent a long time in prison. André, who was one of the founders of the Bourbaki, was in Finland in 1939, officially to visit a few colleagues in the profession, but actually to avoid conscription in his homeland. In World War II, conscription was particularly stupid, as criteria of usefulness were not taken into account as they were considered undemocratic. That led to the absurd situation in which magnificent academic minds were used as simple cannon fodder when practising cryptography and magnetism, for example, could have positively helped the war effort. Also, Weil was a pacifist, but that is irrelevant here. Due to fortuitous circumstances, Weil was wandering alone one sunny day in Finland, which a few days before had become at war with its neighbour Russia. With the war at its height, Weil entertained himself by scrutinising the mysteries of Finnish weaponry and, as you have probably guessed, he

was mistaken for a foreign spy. To top it all off, in his luggage they found some papers from Russian colleagues which alluded to a mysterious man named Bourbaki. Weil argued in vain that he was a mathematician and that he knew their countryman Rolf Nevanlinna (1895-1980). To cut a long story short, he was considered a spy and his execution was set for the next day. That night, and by complete coincidence, the chief of police was at a reception with Nevanlinna, who, by the way, was a colonel in the army reserve. Nevanlinna was horrified when he heard the police explain the terrible luck that had befallen a Mr Weil, along with accompanying phrases like: "And to top it off, the repugnant spy presumes to know you!" In short, Weil reached the safety of the border with Sweden soon after.

But the story does not end there: Weil got to England and he was imprisoned again, ipso facto, for trying to avoid armed services in France – World War II was now raging. He was sent to his homeland, sent to jail once more and, to his disgust (he was happy enough working in the solitude of his cell) he was enlisted again and sent to fight. The French court had taken mercy upon him.

— Norwegian Sophus Lie (1842-1899), father of the Lie group, was a somewhat peculiar character with a bizarre existence. Upon confirming his vocation as a mathematician by resolving a problem, he informed his friend Motzfeld, by waking him up in the middle of the night shouting "I found it! And it is quite simple!" He was also mistaken for a spy, but by the French in the Franco-Prussian War of 1870-1871. It was not easy saving his bacon, as the circumstances of the capture only complicated matters. He was passing the fortifications at Fontainebleau, and oddly it occurred to him to take notes on the landscape. His mathematical notes were interpreted as sophisticated secret codes. Gaston Darboux (1842–1917) had to intervene in order to free Lie from prison and, we assume, from a worse fate still, as France had just lost the bloody war and feelings were running a little high.

— The strangest case of imprisonment of a mathematician is perhaps that of Frenchman André Bloch (1893–1948), who lent his name to a functional space (Bloch space) and a theorem (Bloch theorem). He was certainly intelligent, as apart from his scientific contributions he successfully hid his Jewish origins from the Nazis and continued to publish articles under pseudonyms. Quite an achievement, considering Bloch spent most of his life in a mental institute.

> Bloch was a double candidate for the holocaust, both Jewish and mentally ill. He fought in World War I as an artillery officer and, in 1917, he killed his brother in front of witnesses, explaining to his interrogators that he had done it exclusively for eugenic reasons as his brother could not have continued to live as the carrier of potential mental illnesses. Bloch was declared mentally ill himself and from the quiet of the madhouse he spent 31 years writing excellent articles and corresponding with world-class mathematicians, such as Élie Cartan and Jacques Hadamard. He never came out of his reclusion.

A problem well worth a goose

A goose becoming national news is remarkable on its own. But the awarding of a live goose in exchange for solving a mathematical problem to become an event worthy of national transmission, is even more unusual – but that is what happened in Poland in about 1972. Let's take a look at the details.

The so-called Polish School of Mathematics enjoys deserved fame, and even without the illustrious heights of its music (Chopin, Rubinstein, Penderecki, etc.), it boasts truly great international prestige. Kuratowski, Tarski, Sierpinski and Banach are all illustrious names in the world of mathematics and all university mathematicians know them well.

In the 1930s, just before the great storm of World War II, Polish mathematicians (and a few visitors) in the city of Lviv (now in Ukraine) used to meet in the Scottish Café in the afternoons to discuss matters of interest to them, above all to propose mathematical problems to one another. Some were solved and others were not, as sometimes they overlooked the actual difficulty of the proposals. Occasionally there was a prize for the solution, which was often a bottle of booze and for special occasions it was an object of greater worth. Like a plump and appetising live goose or a mouth-watering feast.

The problems proposed were rigorously recorded in a notebook, which would become the famous 'Scottish Book'. It was available to everyone and kept by the head waiter; little could he have known that in his hands he had some of the most ingenious questions proposed by the human mind.

Among the 37 signatories of the 153 problems in the book were some of the most important minds of the time, such as Stefan Banach (1892–1945), Stanisław Ulam (1909–1984), Kazimierz Kuratowski (1896–1980), Mark Kac (1914–1984), Wacław Sierpinski (1882–1969), Hugo Steinhaus (1887–1972), Samuel Eilenberg (1913–

1998), John von Neumann, Maurice Fréchet (1878–1973), Alexandr Alexandrov (1912–1999) and many others, some of them subsequently executed or murdered (with commendable impartiality) by Russians or Nazis. One famous problem was submitted by Stanisław Mazur (1905–1981) and solved in 1972 by Per Enflo (b. 1944); it was a difficult subject, the prize was a live goose and the prize-giving ceremony was transmitted to the whole country.

It is not known if the communist government's motivation was boosting the morale of the people through the glorification of mathematics or the lack of news, which was typical during the mundane membership of the Soviet paradise. Today, the book's content can be found online and it has been translated into English. And it still contains some unsolved questions.

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The Secret Life of Numbers

Mathematical Curiosities

At the heart of a great deal of serious mathematics lie many mind-boggling anecdotes and crazy curiosities. Mathematicians are famously perceived as eccentric, and this book does nothing to dispel this popular image! What it does do, however, is to uncover the hidden side of mathematics, which is full of absorbing scientific insights, and is fascinating to unravel.